UNCLASSIFIED

AD 93758

Armed Services Technical Information Agency

Reproduced by

DOCUMENT SERVICE CENTER

KNOTT BUILDING, DAYTON, 2, OHIO

UNCLASSIFIED

THEORETICAL SOLUTIONS FOR A STEADY LAMINAR FLOW OF A COMPRESSIBLE FLUID IN THE ENTRANCE REGION OF A TUBE

by

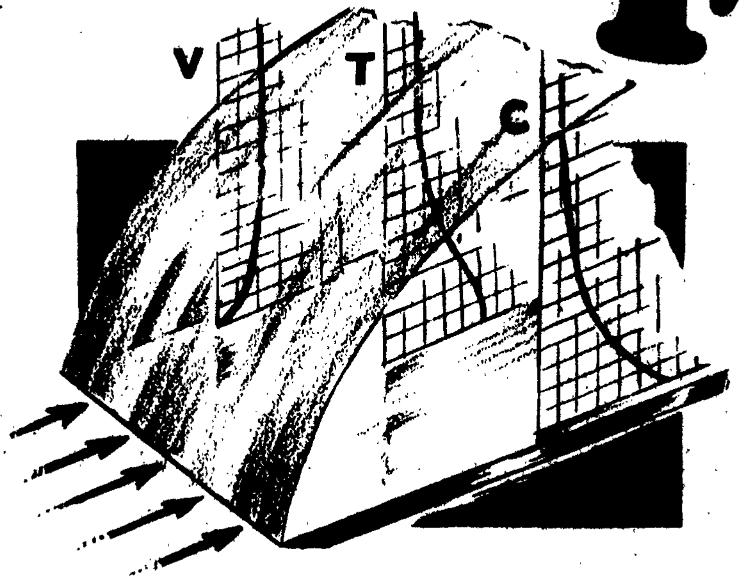
TAU-YI TOONG and JOSEPH KAYE

MARCH 15, 1956

for

OFFICE OF NAVAL RESEARCH, CONTRACT N5ori-07897

FC



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE · 39 · MASSACHUSETTS

DEPARTMENT OF MECHANICAL ENGINEERING
DIVISION OF INDUSTRIAL COOPERATION

TECHNICAL REPORT NO. 7359-11

THEORETICAL SOLUTIONS FOR A STEADY LAMINAR FLOW OF A COMPRESSIBLE FLUID IN THE ENTRANCE REGION OF A TUBE

BY

TAU-YI TOONG AND JOSEPH KAYE

FOR

OFFICE OF NAVAL RESEARCH

CONTRACT N5-ori-07897

D.I.C. PROJECTS 6418 & 7359

MARCH 15, 1956

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE · 39 · MASSACHUSETTS

DEPARTMENT OF MECHANICAL ENGINEERING

DIVISION OF INDUSTRIAL COOPERATION

TABLE OF CONTENTS

LIST OF FIGURES

THEORETICAL SOLUTIONS FOR A STEADY LAMINAR FLOW OF A COMPRESSIBLE FLUID IN THE ENTRANCE REGION OF A TUBE*

by

Tau-Yi Toong** and Joseph Kaye***

Massachusetts Institute of Technology

SUMMARY

The boundary-layer equations of continuity, momentum, and energy for the steady, laminar flow of a compressible fluid in the entrance region of a round tube are solved for the special case of constant values of fluid viscosity and thermal conductivity. These are transformed into a series of ordinary differential equations and then solved with the aid of the Whirlwind Digital Computer for selected entrance and thermal-boundary conditions.

The calculated results presented in the main text correspond to two cases at an entrance Mach number of 2.8, with and without heat transfer. They include recovery factor, friction coefficient, mean pressure-gradient coefficient, heat-transfer coefficient, displacement and momentum thicknesses, and profiles of stagnation temperature, shear stress and heat-transfer rate.

The results for tube flow are compared with those for plate flow with corresponding free-stream and thermal boundary conditions. Near the tube inlet, the plate-flow solution is a good approximation to the tube-flow solution.

---

*This investigation is sponsored as Contract Number N5-ori-07897 by the Office of Naval Research of the United States Navy. Availability of Digital Computer Laboratory time for this problem was made possible also by the Office of Naval Research.

**Associate Professor of Mechanical Engineering.

***Professor of Mechanical Engineering.

NOMENCLATURE

a	inside radius of tube
C	constant, see eq. (1)
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
D	inside diameter of tube
F_m	function of η, see eqs. (15) and (16)
$\underline{f}_m$	function of η, see eq. (7)
$\bar{f}_p$	mean pressure-gradient coefficient, see eq. (A-19)
f_z	local true friction coefficient, see eq. (A-16)
h, i, j, m, n	any positive integer for exponent, subscript, or coefficient
h_z	local heat-transfer coefficient, see eq. (A-21)
K_m	function of η, see eqs. (14), (17), and (18)
k	ratio of specific heats, c_p/c_v
k_m	coefficient of ζ^m in velocity polynomial, see eq. (9)
M	Mach number, $w/(kRT)^{1/2}$
p	static pressure
q	rate of radial heat transfer per unit circumferential area
R	perfect-gas constant
r, φ, z	cylindrical coordinates
$\bar{r}$	recovery factor, see eq. (A-14)
R_D	diameter Reynolds number, $w\,D/\nu$
T	absolute temperature
t_m	coefficient of ζ^m in temperature polynomial, see eq. (11)
u	velocity component in radial direction
w	velocity component in axial direction
z^*	entrance length
α_m	coefficient of ζ^m in density polynomial, see eq. (12)
β_m	coefficient of ζ^m in pressure polynomial, see eq. (10)
δ	boundary-layer thickness
δ^*	displacement thickness, see eq. (A-1)
ζ	modified length Reynolds number, $2(z/DR_{DO})^{1/2}$
η	relative distance ratio, $[1-(r/a)^2]/4\zeta$
ϑ	momentum thickness, see eq. (A-4)
θ_m	function of η, see eq. (8)

λ	thermal conductivity
μ	absolute viscosity
ν	kinematic viscosity
ρ	mass density
σ	Prandtl number, $c_p\mu/\lambda$
τ	shear stress
Ψ	stream function

Subscripts:

aw	refers to adiabatic-wall conditions
c	refers to conditions in isentropic core
0	refers to inlet conditions
s	refers to stagnation conditions
T	refers to temperature-boundary layer
v	refers to velocity-boundary layer
w	refers to conditions at tube wall
∞	refers to free-stream conditions for plate flow
δ	refers to outer edge of boundary layer

Primes denote differentiation with respect to η

INTRODUCTION

The growing importance of problems involving flow of a compressible fluid in ducts with relatively small values of the length-to-diameter ratio has been recognized in many engineering fields during the past decade. In these short ducts, the flow is mainly determined by the nature of the development of the boundary layer in the entrance region, where both the velocity and the temperature profiles change rapidly in the direction of flow.

The present paper presents a theoretical solution for the case of a compressible fluid flowing at either subsonic or supersonic velocities in the entrance region of a round tube, when a laminar boundary layer originates at the tube inlet. This solution is obtained from the integration of the partial differential equations of energy, momentum, and continuity, for selected boundary conditions, for a gas following the perfect-gas rule, and for the special case of constant values of specific heat, viscosity, and thermal conductivity of air. The analysis presented here is not valid when the laminar boundary layer fills the entire cross section of the tube or when the boundary layer becomes turbulent.

The velocity and temperature profiles together with the variation of the stream properties of the isentropic core obtained by the present analysis are reported by Toong and Kaye[1*]. This present paper presents the calculated values of the recovery factor, friction coefficient, mean pressure-gradient coefficient, and heat-transfer coefficient for several specific boundary conditions, in addition to the calculated profiles of stagnation temperature, shear stress, and heat-transfer rate in the laminar boundary layer. The variation of displacement and momentum thicknesses along the length of the tube is also included.

*Superscripts refer to items in Bibliography.

LITERATURE SURVEY

Both analytical and experimental studies of the development of flow in the entrance region of a roundtube are found in the literature. Table 1 summarizes briefly the analytical work for laminar and turbulent flows as well as for compressible and incompressible fluids*.

Most of the theoretical analyses are limited to the case of a laminar flow of an incompressible fluid in a round tube. Hagenbach[2], Neumann[3], and Couette[4] applied merely a correction accounting for the change in the kinetic energy to the solution of a fully developed laminar flow; Schiller[5], and Shapiro, Siegel and Kline[6] used the integral method; Boussinesq[7], Atkinson and Goldstein[8], and Langhaar[9] solved the partial differential equations of momentum and continuity; and Kays[10] solved the energy equation, using the calculated velocity profile of Langhaar.

The agreement between the calculated values of the center-line velocity by Boussinesq[7] and the experimental values of Nikuradse[11] is within 3% for $z/DR_{DO} > 0.02$. However, this difference is increased to about 10% at smaller values of z/DR_{DO}, such as at 0.0075. Atkinson and Goldstein[8] used the method of Boussinesq to extend their series solution. When these two solutions are joined together at $z/DR_{DO} = 0.000625$, the agreement between the theoretical center-line velocity and the measured value of Nikuradse is improved to within 3% for $z/DR_{DO} > 0.0075$. The agreement between the calculated values of the velocity profile across the tube section and the measured values of Nikuradse is within 10% for $z/DR_{DO} > 0.0075$.**

The agreement between the theoretical value of the center-line velocity of Langhaar[9] and the experimental value of Nikuradse[11] is within 10% for the entire entrance length***; but this agreement improves to better than 3% for $z/DR_{DO} > 0.01$.

---

*Those analyses dealing with diabatic flows with fully developed velocity profiles are not included here.

**See Fig. 81 on p. 308 of Goldstein[8].

***Entrance length obtained by the integral method is defined as the distance at which $\delta/a = 1$, and that obtained from the solution of the boundary-layer equations is defined as the distance where the center-line velocity is 99% of the Poiseuille value.

For small values of z/DR_{DO} where $\delta/a \ll 1$, the velocity profiles assumed in the integral methods of Schiller[5] and Shapiro, Siegel, and Kline[6] agree well with those calculated by the more exact methods of solving the differential equations of momentum and continuity. For example, for $z/DR_{DO} < 0.000625$, the velocity profiles of Schiller[5] agree with those of Atkinson and Goldstein[8] within 1% for $r/a \leq 0.8$ and within 5% for $r/a = 0.9$. However, for such small values of z/DR_{DO}, the experimental results do not agree with the theoretical solutions.*

The values of the center-line velocity calculated by Schiller agree within 10% for the entire entrance length with the values measured by Nikuradse. The agreement gets poorer, however, as z/DR_{DO} approaches the limiting value corresponding to the entrance length, in contrast to the improved agreement observed between the velocities calculated from the solution of the boundary-layer equations and the measured values.

The entrance lengths calculated by the integral methods of Schiller[5] and Shapiro et al.[6] are about 30% smaller than the experimental value ($0.043\ DR_{DO}$) of Nikuradse[11]. The entrance lengths obtained by Boussinesq[7], Atkinson and Goldstein[8], and Langhaar[9] are higher than the Nikuradse's value by 50%, 26%, and 33%, respectively.

The mean pressure-gradient coefficient at a distance from the tube inlet greater than the entrance length is calculated from an equation of the following form:

$$2\,(p_0 - p)\,/\rho_0\, w_0^2 = C + 64\, z/DR_{DO} \tag{1}$$

The values of C in eq. (1) obtained by Boussinesq[7], Atkinson and Goldstein[8], and Langhaar[9] from the solutions of the boundary-layer equations agree within 7% with the mean experimental value (1.32) of Schiller[12]. The values calculated by the integral methods of Schiller[5] and Shapiro et al.[6] and by the method of kinetic-energy correction are lower by 12%, 18%, and 24%, respectively, than the experimental value of Schiller[12]. However, it is important to note also that the difference between the mean experimental values of Schiller[12] and those of Riemann[13] is about 6%.

*See Fig. 80 on p. 304 of Goldstein[8].

A comparison of the theoretical and experimental values of the mean pressure-gradient coefficient at a distance from the tube inlet smaller than the entrance length may be found in the work of Shapiro et al.[6] Over the range of the experimental values of $10^{-5} < z/DR_{DO} < 10^{-3}$, obtained by Kline and Shapiro[14], the calculated values of Schiller[5] and Shapiro et al.[6] agree within 6% and 2%, respectively, of the experimental values. For $z/DR_{DO} > 0.0075$, a maximum difference of 7% is observed between the calculated values of Atkinson and Goldstein[8] and Shapiro et al[6]. As z/DR_{DO} increases toward its limiting value corresponding to the entrance length, this difference is reduced. Fig. 5 of reference (6) shows that, for $z/DR_{DO} > 0.0075$, the mean pressure-gradient coefficients calculated by Atkinson and Goldstein are always greater than those calculated by Shapiro et al., with those calculated by Langhaar[9] lying between them.

As z/DR_{DO} approaches zero, the experiments of Kline and Shapiro[14] show that the ratio of the mean pressure-gradient coefficient to z/DR_{DO} approaches an asymptotic value of 13.74. The corresponding asymptotic values obtained from the analyses of Atkinson and Goldstein[8], Shapiro et al.[6] and Schiller[5] are 0.2%, 2%, and 6% higher, respectively, than the experimental value. The solution of Langhaar[9] does not seem to have the similar asymototic behavior near the tube inlet as others.

For the laminar flow of a compressible fluid in the entrance region of a circular tube, only the theoretical analyses of Toong[15] and Toong and Shapiro[16] are available. Toong[15] solved the partial differential equations of continuity, momentum, and energy, for selected boundary conditions and for the special case of constant values of specific heat, viscosity, and thermal conductivity of air. Toong and Shapiro[16] used an integral method, with the values of the viscosity and thermal conductivity of air, which either varied according to the Sutherland's equation or were constant.

As for the case of an incompressible fluid, the integral method gives valid results only when the boundary-layer thickness is small as compared with the tube radius. In the region where $\delta/a < 0.3$, the agreement between the calculated values of the mean pressure-gradient coefficient by the two analyses quoted above is within 5%.

The theoretical analysis of Latzko[17] is available in the study of the turbulent flow of an incompressible fluid in the entrance region of a round tube. In this analysis, the developing turbulent boundary layer is assumed to originate at the tube inlet, with the thickness of the velocity-boundary layer equal to that of the temperature-boundary layer.

PROBLEM

The flow model of the present paper is shown in Fig. 1. A compressible fluid flows steadily in the entrance region of a round tube. The velocity and temperature profiles at the tube inlet, or station 0, are assumed to be uniform. A laminar boundary layer is assumed to grow in thickness from the inlet station until at some point downstream the laminar boundary layer either fills the entire cross section of the tube or becomes turbulent. The analysis presented in this paper is not valid in the region downstream of such a point.

The following assumptions are used in the analysis:

1. The fluid flow inside the core is one-dimensional and isentropic.
2. Air is a perfect gas with constant specific heats.
3. Negligible external body forces exist.
4. The flow is rotationally symmetric about the tube axis.
5. The usual assumptions for boundary-layer problems are also made; viz.,

$$\frac{u}{w} \ll 1,$$

$$\frac{\delta_v}{z} \ll 1,$$

and since $c_p \mu/\lambda = O(1)$ for most gases,

$$\frac{\delta_T}{z} \ll 1$$

ANALYSIS

The analysis of the present problem is given in the earlier paper of Toong and Kaye[1]. Only a brief outline will be given below. A summary of some important physical quantities is listed in Appendix A.

By the use of the above assumptions, the basic equations of continuity, momentum, and energy are first transformed into the following equations for the boundary-layer flow:

$$\frac{\partial}{\partial r}(r\rho u) + \frac{\partial}{\partial z}(r\rho w) = 0 \qquad (2)$$

$$\frac{\partial p}{\partial r} = 0 \qquad (3)$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial w}{\partial r}\right) \qquad (4)$$

$$\rho c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = w\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial T}{\partial r}\right) + \mu\left(\frac{\partial w}{\partial r}\right)^2 \qquad (5)$$

Next, the partial differential equations (2), (3), (4), and (5) are transformed into a series of ordinary differential equations by means of the following assumptions:

1. A stream function Ψ, satisfying the continuity equation (2) such that

$$\left.\begin{aligned} \frac{\partial \Psi}{\partial r} &= \rho w\, r \\ &\text{and} \\ \frac{\partial \Psi}{\partial z} &= -\rho u\, r, \end{aligned}\right\} \qquad (6)$$

can be expressed in terms of the following convergent series:

$$\Psi = -a^2 \rho_0 w_0 \sum \xi^m f_m(\eta) \qquad (7)$$

where ξ, the modified length Reynolds number, and η, the relative distance ratio, are the two new independent variables, and are defined in Nomenclature.

2. The temperature in the boundary layer can be expressed in terms of the following convergent series:

$$T = T_0 \sum \xi^{m-1} \theta_m(\eta) \qquad (8)$$

3. The velocity and the properties of the fluid in the isentropic core are expressed in terms of the following series, whose coefficients may be determined from the conditions of steady flow[15]:

$$w_c = w_0 \quad (1 + \sum \xi^m k_m) \tag{9}$$

$$p = p_0 \quad (1 + \sum \xi^m \beta_m) \tag{10}$$

$$T_c = T_0 \quad (1 + \sum \xi^m t_m) \tag{11}$$

$$\rho_c = \rho_0 \quad (1 + \sum \xi^m \alpha_m) \tag{12}$$

For the special case of constant values of viscosity and thermal conductivity of the compressible fluid, the final transformed equations are as follows:

$$\left.\begin{aligned} & F_1'' + f_1 \, F_1' = 0 \\ & (1/\sigma) \;\; \theta_1'' + f_1 \;\; \theta_1' + (k-1) \, M_0^2 \;\; (F_1')^2 \, / \, 4 = 0 \end{aligned}\right\} \tag{13}$$

and

$$\left.\begin{aligned} & F_m'' + f_1 \, F_m' - (m-1) \;\; f_1' \;\; F_m + mF_1' \;\; f_m \;=\; K_{2m-3} \, (\eta) \\ & (1/\sigma) \; \theta_m'' + f_1 \; \theta_m' - (m-1) \; f_1' \;\; \theta_m + m\theta_1' \;\; f_m + (k-1) \; M_0^2 \; F_1' \;\; F_m' \, /2 \\ & \qquad = \; K_{2m-2} \quad (\eta) \end{aligned}\right\} \tag{14}$$

where

$$F_1 \equiv \theta_1 f_1' \tag{15}$$

$$F_m \equiv \theta_1 \; f_m' + f_1' \;\; \theta_m, \text{ for } m \geqslant 2 \tag{16}$$

and $K_{2m-3}(\eta)$ and $K_{2m-2}(\eta)$ are known functions of η; for example,

$$K_1(\eta) = 4\beta_1 / kM_0^{\,2} + 4F_1' \quad -\beta_1 f_1' F_1 - 4\eta f_1 F_1' \qquad (17)$$

$$K_2(\eta) = 4\theta_1'/\sigma + (k-1) M_0^{\,2}\beta_1 (F_1')^2/2 - (k-1)\beta_1 F_1/k - 4\eta f_1 \theta_1' \qquad (18)$$

Boundary Conditions

At the outer edge of the boundary layer, provided that an isentropic core exists, it may be shown that[15]

$$(f_m')_c = 2\sum_{i=0}^{m-1} k_i \alpha_{m-i-1} \qquad (19)$$

and

$$(\theta_m)_c = t_{m-1} \qquad (20)$$

with $k_0 = \alpha_0 = t_0 = 1$, $m = 1, 2, \ldots$.

The condition that the velocity vanish at the tube wall requires that

$$f_m(0) = 0 \qquad (21)$$

$$f_m'(0) = 0 \qquad (22)$$

The condition of an adiabatic wall requires that

$$\theta_m'(0) = 0 \qquad (23)$$

and the condition of a wall maintained at an arbitrary temperature distribution requires that

$$\sum \xi^{m-1} \theta_m(0) = \theta_w \qquad (24)$$

RESULTS

The transformed boundary-layer equations (13) and (14) for the special case of constant values of viscosity and thermal conductivity of air were solved first with the aid of the M.I.T. Differential Analyzer and more recently with the aid of the Whirlwind I Digital Computer for better accuracy. The solutions obtained from the Whirlwind Computer agree with those obtained from the Differential Analyzer to the last significant figure of the latter solutions. The results reported in this paper are obtained from the Whirlwind Computer and are tabulated in Appendix B.

Several solutions of the equations were obtained for different sets of entrance and thermal-boundary conditions, but only two cases are presented in this section. Both of these cases correspond to an entrance Mach number of 2.8; case A is for adiabatic flow and case C is for flow with heat transfer to the compressible fluid at $\Theta_w = 3$. The velocity and temperature profiles together with the variation of the stream properties in the isentropic core corresponding to these two cases have been reported by Toong and Kaye[1]. Calculated values of recovery factor, friction coefficient, mean pressure-gradient coefficient, heat-transfer coefficient, displacement and momentum thicknesses, and profiles of stagnation temperature, shear stress, and heat-transfer rate are given here. These physical quantities can be computed from the equations given in Appendix A.

Boundary-Layer Thickness-Ratios

The boundary-layer thickness δ_v or δ_T is related through eq. (A-3) in Appendix A to the corresponding value of the relative distance ratio η at which the velocity or temperature inside the boundary layer reaches a certain arbitrary percentage of the core velocity or temperature, respectively. The upper part of Fig. 2 shows the variation of the thickness ratios for both the velocity-and temperature-boundary layers, when the first terms of the series solutions for the velocity and temperature in the boundary layer are equal to 99% of the core velocity and core temperature, respectively.

Fig. 2 indicates that the thickness of the velocity-boundary layer is smaller than that of the temperature-boundary layer, for given values of the entrance Mach number and given thermal conditions at the tube wall. This result is also predicted from a study of the basic equations of motion; i.e., the ratio of the thickness of the velocity-boundary layer to that of the temperature-boundary layer is shown to be of the order of the square root of the Prandtl number of the fluid flowing inside the tube.

For given values of the entrance Mach number, the effect of heating the fluid at the tube wall is to increase the boundary-layer thicknesses.

Shown for comparison in the upper part of Fig. 2 are also the boundary-layer thickness-ratios predicted for an adiabatic laminar flow of a compressible fluid over a flat plate, with the free-stream conditions identical with the entrance conditions of the corresponding tube flow. Near the tube inlet, it is observed that the plate-flow solution is a good approximation to the tube-flow solution.

The middle part of Fig. 2 shows the variation of the displacement and momentum thicknesses, defined by eqs. (A-1) and (A-4), for tube flows at $M_0 = 2.8$ with and without heat transfer. Two dashed curves are also included to show the variation of these thicknesses for an adiabatic laminar flow over a flat plate at a free-stream Mach number of 2.8. Near the tube inlet, the curves for tube flow are almost linear with respect to the modified length Reynolds number, thus demonstrating again the characteristics of a plate flow.

The ratio of displacement thickness to momentum thickness is shown in the lower part of Fig. 2. For a given entrance Mach number and thermal condition at the tube wall, this ratio is almost independent of the modified length Reynolds number. Shown for comparison are also two curves for laminar flow over a flat plate at $M_\infty = 2.8$, with and without heat transfer. For plate flows, the ratio of these thicknesses remains constant.

Region of Validity of Present Analysis

Since the present analysis is valid only when the boundary layer does not fill the entire cross section of the tube, the calculated results should not be extended beyond a certain maximum value of the modified length Reynolds number. Since the present analysis is valid only when the boundary layer remains laminar, the calculated results should not be extended beyond a certain value of the transitional length Reynolds number. Fig. 3 is constructed, on the basis of these limitations, to show the region of validity of the present analysis for an entrance Mach number of 2.8. Two values are selected for both the maximum modified length Reynolds number and the transitional length Reynolds number. The values of the length-to-diameter ratios below the straight line of a given modified length Reynolds number and the rectangular hyperbola of a given transitional length Reynolds number are in the valid region of the present analysis. It is observed that the limiting condition at low entrance diameter Reynolds numbers is the value of the modified length Reynolds number, while the limiting condition at high entrance diameter Reynolds numbers is the value of the transitional length Reynolds number.

Profiles of T_s/T_{sc}, τ/τ_w, $qD/\lambda_0 T_0$, and q/q_w

Fig. 4 presents for the adiabatic flow at $M_0 = 2.8$ the profiles of the stagnation temperature, shear stress, and heat-transfer rate across the laminar boundary layer at three different axial positions. They are calculated from eqs. (A-6), (A-11), and (A-12), respectively. The first two profiles for T_s/T_{sc} and τ/τ_w are shown at $\zeta = 0$, 0.03, and 0.05, and the third profile for $qD/\lambda_0 T_0$ is shown at $\zeta = 0.01$, 0.03, and 0.05.

Fig. 5 presents the profiles of the stagnation temperature, shear stress, and heat-transfer rate for flow with heat transfer to the compressible fluid at $\theta_w = 3$ and $M_0 = 2.8$. They are calculated from eqs. (A-6), (A-11), and (A-13), respectively. Note the difference in the ordinate representing the heat-transfer rate in Figs. 4 and 5. Also note that in Fig. 5, all the three profiles are shown at $\zeta = 0$, 0.03, and 0.05.

Near the outer edge of the boundary layer adjacent to the core, a slight bump is found at the larger values of ζ for the profiles shown in Figs. 4 and 5. Similar bumps are observed in the velocity and temperature profiles in the earlier paper of Toong and Kaye[1]. These bumps have actually been measured for supersonic flow of air in a round tube by Kaye, et al.[18,19]

The bump in the stagnation-temperature profile of Fig. 4 can be checked by evaluating the integral of the enthalpy flux with respect to radius for various sections at $\zeta = 0$, 0.03, and 0.05. These fluxes were found to remain constant within 1%.

It is to be noted that the profiles at $\zeta = 0$ not only show the solution at the tube inlet but also represent the solution of the corresponding plate flow. In fact, for adiabatic flow at $M_0 = 2.8$, the agreement between the tube solution at $\zeta = 0$ and the solution of Emmons and Brainerd[20] is within 1%. Since the solution at $\zeta = 0$ is a first approximation to the series solution of the tube flow at some finite distance from the inlet, it is demonstrated once again that near the tube inlet, the tube flow behaves like a plate flow with corresponding free-stream conditions.

Recovery Factor, Friction Coefficient, Mean Pressure-Gradient Coefficient, and Heat-Transfer Coefficient

Fig. 6 shows the variation of recovery factor, friction coefficient, mean pressure-gradient coefficient, and heat-transfer coefficient with respect to the modified length Reynolds number.

The recovery factor, defined by eq. (A-14) and calculated from eq. (A-15) is plotted at the top of Fig. 6 for the adiabatic flow at $M_0 = 2.8$. The recovery factor increases almost linearly from 0.846 to 0.855 as the value of ζ increases from zero to 0.04.

The theoretical analysis of Emmons and Brainerd[20] for an adiabatic laminar flow of a compressible fluid over a flat plate indicates that, as a good first approximation, the recovery factor is equal to the square root of the Prandtl number of the compressible fluid. In Fig. 6, the line representing the square root of the Prandtl number (0.74) used in the present calculation is also shown for comparison. Thus the tube-flow solution near the inlet is found to be one per cent less than the approximate plate-solution.

The local true friction coefficient, defined by eqs. (A-16) and (A-17) and calculated from eq. (A-18), is shown in Fig. 6 for flows at $M_0 = 2.8$, with and without heat transfer. The ordinate is in the form of the product of the friction coefficient and the square root of the length Reynolds number. For laminar flow over a flat plate, this product is constant; while for laminar flow in a round tube, this product decreases almost linearly by about 15 to 20% as the value of ζ increases from zero to 0.04. Shown for comparison in Fig. 6 is also the curves for laminar flow over a flat plate at a free-stream Mach number of 2.8, with and without heat transfer. Again, it is observed that near the tube inlet, the plate-flow solution is a good approximation of the tube-flow solution.

Fig. 6 also indicates that the effect of heating is to decrease the friction coefficient for the same entrance diameter Reynolds number and at the same length Reynolds number.

The mean pressure-gradient coefficient, defined according to eq. (A-19) and calculated from eq. (A-20) is shown in Fig. 6 for flows at $M_0 = 2.8$, with and without heat transfer. Here it is observed that the effect of heating the compressible fluid at the tube wall on the mean pressure-gradient coefficient is just opposite to that on the local true friction coefficient; viz., heating gives higher values of the pressure-gradient coefficient but lower values of the friction coefficient. This apparent discrepancy is due to the different ways the two coefficients are defined. The pressure-gradient coefficient is related to the change in static pressure along the length of the tube. Using the knowledge developed in the one-dimensional flow of a compressible fluid inside a duct of constant area that the effects on pressure change due to heating and friction are in the same direction, it seems justified that the value of the pressure-gradient coefficient is increased when the fluid is heated at the tube wall. On the other hand, the friction coefficient is related to the radial velocity gradient at the tube wall. When the fluid is heated, the boundary-layer thickness is increased, the radial velocity gradient at the wall is decreased, and therefore the friction coefficient is decreased.

At the bottom of Fig. 6 is shown a plot of the product of the Stanton number and the square root of the length Reynolds number, for laminar flow with heat transfer at $\theta_w = 3$ and $M_0 = 2.8$. The heat-transfer coefficient is defined by eq. (A-21), and the Stanton number is calculated from eq. (A-22). It is observed that this product is almost independent of the modified length Reynolds number.

In the same plot is shown also a line for the laminar flow of a compressible fluid over a flat plate at $M_\infty = 2.8$ and $T_w/T_\infty = 3$. The two solutions agree within 2%.

BIBLIOGRAPHY

[1]Toong, T. Y. and Kaye, J., Theoretical Velocity and Temperature Profiles for the Laminar Boundary Layer of the Flow of a Compressible Fluid in the Entrance Region of a Tube, Proceedings of the Second U. S. National Congress of Applied Mechanics, pp. 781-785, 1954.

[2]Hagenbach, E., Über die Bestimmung der Zähigkeit einer Flüssigkeit durch den Ausfluss aus Röhren, Poggendorffs Annalen der Physik und Chemie (4), Vol. 109, pp. 385-426, 1860.

[3]Neumann, F. E., Einleitung in die theoretische Physik, pp. 258-262, Leipzig, 1883.

[4]Couette, M., Études sur le frottement des liquides, Annales de Chimie et de Physique (6), Vol. 21, pp. 494-510, 1890.

[5]Schiller, L., Untersuchungen über laminare und turbulente Strömung, Forschungsarbeiten, V.D.I., Vol. 248, pp. 17-25, 1922.

Schiller, L., Die Entwicklung der laminaren Geschwindigkeitsverteilung und ihre Bedeutung für Zähigkeitsmessungen, Zeitschrift für angewandte Mathematik und Mechanik, Vol. 2, pp. 96-106, 1922.

[6]Shapiro, A. H., Siegel, R., and Kline, S. J., Friction Factor in the Laminar Entry Region of a Smooth Tube, Proceedings of the Second U. S. National Congress of Applied Mechanics, pp. 733-741, 1954.

[7]Boussinesq, J., Sur la manière dont les vitesses, dans un tube cylindrique de section circulaire, évasé à son entrée, se distribuent depuis cette entrée jusqu'aux endroits où se trouve établi un régime uniforme, Comptes Rendus, Vol. 113, pp. 9-15, 1891.

Boussinesq, J., Calcul de la moindre longueur que doit avoir un tube circulaire, évasé à son entrée, pour qu'un régime sensiblement uniforme s'y établisse, et de la dépense de charge qu'y entraîne l'établissement de ce régime, Comptes Rendus, Vol. 113, pp. 49-51, 1891.

[8]Goldstein, S., Modern Developments in Fluid Dynamics, Vol. 1, pp. 304-308, Oxford University Press, London, England, 1938.

[9]Langhaar, H. L., Steady Flow in the Transition Length of a Straight Tube, Journal of Applied Mechanics, Trans. ASME, Vol. 64, pp. A-55--A-58, 1942.

[10]Kays, W. M., Numerical Solutions for Laminar Flow Heat Transfer in Circular Tubes, Technical Report No. 20, Navy Contract N6ONR-251, T.O. 6, Mech. Eng. Dept., Stanford University, Oct. 1953.

[11]Prandtl, L. and Tietjens, O. G., Applied Hydro-and Aeromechanics, p. 27, McGraw-Hill Book Co., Inc., New York, 1934.

[12]Schiller, L., Handbuch der Experimentalphysik, IV, 4 Teil, Hydro-und Aerodynamik, p. 57, Akademische Verlagsgesellschaft M.B.H., Leipzig, Germany, 1932.

[13]Riemann, W., 3rd., The Value of the Hagenbach Factor in the Determination of Viscosity by the Efflux Method, Journal of the American Chemical Society, Vol. 50, p. 46, 1928.

[14]Kline, S. J. and Shapiro, A. H., The Effect of Cooling on Boundary Layer Transition in a Gas, Final Report of DIC Project 3-6927 at Massachusetts Institute of Technology, Sept., 1952.

[15]Toong, T. Y., The Laminar Boundary Layer of a Steady, Compressible Flow in the Entrance Region of a Tube, Sc.D. Thesis, Mechanical Engineering Department, Massachusetts Institute of Technology, Jan. 1952.

[16]Toong, T. Y. and Shapiro, A. H., Theoretical Investigation of Frictional Effects for Laminar Compressible Flow in a Tube Entry, 50 Jahre Grenzschichtforschung, pp. 181-192, Friedr. Vieweg & Sohn, Braunschweig, 1955.

[17]Latzko, H., Der Wärmeübergang an einen turbulenten Flüssigkeits-oder Gasstrom, Zeitschrift für angewandte Mathematik und Mechanik, Vol. 1, pp. 277-284, 1921, or NACA TM1068, 1944.

[18]Kaye, J., Brown, G. A., Dieckmann, J. J. and Sziklas, E. A., Experimental Velocity Profiles for Supersonic Flow of Air in a Tube with and without Heat Transfer, Proceedings of the Second U. S. National Congress of Applied Mechanics, pp. 787-792, 1954.

[19]Kaye, J., Brown, G. A., Westmoreland, J. C., and Kurtz, E. F., Jr., Simultaneous Measurements of Velocity and Temperature Profiles for Adiabatic Supersonic Flow of Air in a Tube, Journal of the Aeronautical Sciences, Vol. 22, No. 8, pp. 575-577, 1955.

[20]Emmons, H. W., and Brainerd, J. G., Temperature Effects in a Laminar Compressible-Fluid Boundary Layer Along a Flat Plate, Journal of Applied Mechanics, Trans. ASME, Vol. 63, pp. A-105-A-110, 1941.

TABLE I

SUMMARY OF THEORETICAL ANALYSES FOR FLOW IN THE ENTRANCE REGION OF A ROUND TUBE

Author	Year	Flow Type	Vel'y Profile w/w_c	Temp. Profile T/T_c	Entrance Length, z^*	Mean Pres.-Grad. Coef., $4\bar{f}_p(z/D)$ $z < z^*$	Mean Pres.-Grad. Coef., $4\bar{f}_p(z/D)$ $z > z^*$	Friction Coef., f_z	Recovery Factor, $\bar{r}$	Heat-Transfer Coef., h_z	Method of Analysis
Hagenbach[2]	1860	Incomp., Laminar	---	---	---	---	$1+64z/DR_{DO}$ †	---	---	---	Kinetic-energy correction
Neumann[3]	1860	Incomp., Laminar	---	---	---	---	$1+64z/DR_{DO}$	---	---	---	Kinetic-energy correction
Couette[4]	1890	Incomp., Laminar	---	---	---	---	$1+64z/DR_{DO}$	---	---	---	Kinetic-energy correction
Boussinesq[7]	1891	Incomp., Laminar	Calc'd	---	$0.065DR_{DO}$	Calc'd	$1.24+64z/DR_{DO}$	Calc'd	---	---	Boundary-layer equations
Latzko[17]	1921	Incomp., Turbulent	Assumed	Calc'd	$.625D(R_{DO})^{1/4}$	Calc'd	---	Calc'd	---	Calc'd	Integral method
Schiller[5]	1922	Incomp., Laminar	Assumed	---	$.0288DR_{DO}$	Calc'd	$1.16+64z/DR_{DO}$	---	---	---	Integral method
Atkinson and Goldstein[8]	before 1938	Incomp., Laminar	Calc'd	---	$.054DR_{DO}$	Calc'd	$1.41+64z/DR_{DO}$	Calc'd	---	---	Boundary-layer equations
Langhaar[9]	1941	Incomp., Laminar	Calc'd	---	$.057DR_{DO}$	Calc'd	$1.28+64z/DR_{DO}$	Calc'd	---	---	Boundary-layer equations
Toong[15]	1952	Compres., Laminar	Calc'd	Calc'd	---	Calc'd	---	Calc'd	Calc'd	Calc'd	Boundary-layer equations
Kays[10]	1953	Incomp., Laminar	Assumed	Calc'd	---	---	---	---	---	Calc'd	Boundary-layer equations
Toong and Kaye[1]	1954	Compres., Laminar	Calc'd	Calc'd	---	Calc'd	---	---	---	---	Boundary-layer equations
Shapiro, Siegel and Kline[6]	1954	Incomp., Laminar	Assumed	---	$.0300DR_{DO}$ for modified cubic vel'y profile $.0296DR_{DO}$ for modified Pohlhausen vel'y profile	Calc'd	$1.080+64z/DR_{DO}$ for modified cubic vel'y profile $1.106+64z/DR_{DO}$ for modified Pohlhausen vel'y profile	---	---	---	Integral method
Toong and Shapiro[16]	1954	Compres., Laminar	Assumed	Assumed	---	Calc'd	---	---	---	---	Integral method

†Author's error in the derivation corrected.

APPENDIX A

Summary of Important Physical Quantities

1. Displacement Thickness, δ^*

$$\rho_c w_c \pi (a - \delta^*)^2 \equiv \rho_c w_c \pi a^2 - 2\pi \int_{a-\delta}^{a} (\rho_c w_c - \rho w) \, r dr \qquad \text{(A-1)}$$

$$\text{or } 2\left(\frac{\delta^*}{a}\right) - \left(\frac{\delta^*}{a}\right)^2 = 4\zeta \int_0^{\eta_\delta} \left(1 - \frac{\rho w}{\rho_c w_c}\right) d\eta \qquad \text{(A-2)}$$

$$\text{where } \eta_\delta \equiv \left[1 - \left(1 - \frac{\delta}{a}\right)^2\right] / 4\zeta \qquad \text{(A-3)}$$

2. Momentum Thickness, ϑ

$$\rho_c w_c^2 \pi (a - \vartheta)^2 \equiv \rho_c w_c^2 \pi a^2 - 2\pi \int_{a-\delta}^{a} (w_c - w) \rho w \, r dr \qquad \text{(A-4)}$$

$$\text{or } 2\left(\frac{\vartheta}{a}\right) - \left(\frac{\vartheta}{a}\right)^2 = 4\zeta \int_0^{\eta_\delta} \frac{\rho w}{\rho_c w_c} \left(1 - \frac{w}{w_c}\right) d\eta \qquad \text{(A-5)}$$

3. Stagnation-Temperature Profile, T_s/T_{sc}

$$\frac{T_s}{T_{sc}} = \frac{T_s}{T_{s0}} = T\left(1 + \frac{k-1}{2} M^2\right) / T_0\left(1 + \frac{k-1}{2} M_0^2\right) \qquad \text{(A-6)}$$

$$\text{with } \frac{M^2}{M_0^2} = \left(\frac{w_c}{w_0}\right)^2 \left(\frac{w}{w_c}\right)^2 \left(\frac{T_0}{T}\right) \qquad \text{(A-7)}$$

$$\frac{w_c}{w_0} = 1 + \sum \zeta^m k_m \qquad \text{(A-8)}$$

$$\frac{w}{w_c} = \frac{\sum \zeta^{m+n-2} \theta_m f_n'}{2(1 + \sum \zeta^h \alpha_h)(1 + \sum \zeta^i t_i)(1 + \sum \zeta^j k_j)} \qquad \text{(A-9)}$$

$$\text{and } \frac{T}{T_0} = \sum \zeta^{m-1} \theta_m \qquad \text{(A-10)}$$

4. Shear-Stress Profile, τ/τ_w

$$\frac{\tau}{\tau_w} = \left(\frac{\partial w}{\partial r}\right) \Big/ \left(\frac{\partial w}{\partial r}\right)_w$$

$$= \frac{r \sum \zeta^{m+n-2} (\theta_m f_n'' + \theta_m' f_n')}{a \sum \zeta^{m+n-2} (\theta_m f_n'' + \theta_m' f_n')_w} \qquad \text{(A-11)}$$

5. Heat-Transfer-Rate Profile, $q\, D/\lambda_0 T_0$ or q/q_w

$$\frac{q\, D}{\lambda_0\, T_0} = -\frac{r}{a} \left(\sum \zeta^{m-2}\, \theta_m' \right) \qquad \text{(A-12)}$$

and $$\frac{q}{q_w} = \frac{r \left(\sum \zeta^{m-1}\, \theta_m' \right)}{a \left(\sum \zeta^{m-1}\, \theta_m' \right)_w} \qquad \text{(A-13)}$$

6. Recovery Factor, $\bar{r}$

$$\bar{r} \equiv (T_{aw} - T_c) \,/\, (T_{sc} - T_c) \qquad \text{(A-14)}$$

or $$\bar{r} = \frac{\sum \zeta^{m-1}\, \theta_m(0) - (1 + \sum \zeta^n t_n)}{(k-1)\, M_0^2/2 - \sum \zeta^n t_n} \qquad \text{(A-15)}$$

7. Local True Friction Coefficient, f_z

$$f_z \equiv \tau_w / \left(\tfrac{1}{2} \quad \rho_0 w_0^2\right) \qquad \text{(A-16)}$$

with $$\tau_w = -\mu \left(\frac{\partial w}{\partial r}\right)_w \qquad \text{(A-17)}$$

or $$f_z = \frac{(\nu_0/w_0 z)^{1/2} \sum \zeta^{m+n-2} (\theta_m f_n'' + \theta_m' f_n')_w}{2 \left(1 + \sum \zeta^m \beta_m \right)} \qquad \text{(A-18)}$$

8. Mean Pressure-Gradient Coefficient, $4\ \bar{f}_p\ (z/D)$

$$4\ \bar{f}_p\ (z/D) \equiv (p - p_0) \Big/ \tfrac{1}{2}\ \rho_0\ w_0^2 \tag{A-19}$$

or $$4\ \bar{f}_p\ (z/D) = 2\left(\sum \zeta^n \beta_n - 1\right) / k\ M_0^2 \tag{A-20}$$

9. Local Heat-Transfer Coefficient, h_z

$$h_z \equiv q/(T_w - T_{aw}) \tag{A-21}$$

or $$\frac{h_z}{c_p \rho_0 w_0} =$$

$$- \frac{1}{2\sigma} \left(\frac{\nu_0}{w_0 z}\right)^{1/2} \sum \zeta^{m-1} (\theta_m')_w \Big/ \sum \zeta^{m-1} \left[(\theta_m)_w - (\theta_m)_{aw} \right] \tag{A-22}$$

APPENDIX B

TABULATION OF RESULTS

The transformed boundary-layer equations (13) and (14) for the special case of constant values of fluid viscosity and thermal conductivity were solved with the aid of the Whirlwind I Digital Computer for the six cases shown below in Table B-1. The Prandtl number of the fluid was assumed constant at 0.74. The series solution of each of these cases was carried up to terms containing ξ^{m-1}, with the value of m indicated in the table.

TABLE B-1

Case	Entrance Mach Number	Thermal Condition at Tube Wall	m
A	2.8	Adiabatic	3
B	2.8	Cooling, $\theta_w = 2$	2
C	2.8	Heating, $\theta_w = 3$	3
D	2	Adiabatic	2
E	2	Cooling, $\theta_w = 1.445$	2
F	2	Heating, $\theta_w = 2.167$	2

The functions f_n , F_n , θ_n , and their derivatives are tabulated in Table B-2, and the quantities β_n , t_n , α_n, and k_n in Table B-3. The physical quantities listed in Appendix A can be computed from these tables.

TABLE B-2

Values of f_n, F_n, F_n', θ_n, and θ_n'

(i) Case A, $M_0 = 2.8$, Adiabatic

η	f_1	F_1	F_1'	θ_1	$-\theta_1'$
0.0	0.00000	0.00000	0.96779	2.3269	0.00000
.1	. 208	. 9678	.96772	2.3242	. 5433
.2	. 834	.19353	.96725	2.3160	.10861
.3	. 1881	.29020	.96597	2.3025	.16269
.4	. 3357	.38668	.96348	2.2835	.21633
.5	. 5272	.48284	.95937	2.2592	.26920
.6	. 7636	.57849	.95323	2.2297	.32087
.7	.10466	.67341	.94468	2.1951	.37082
.8	.13778	.76733	.93334	2.1556	.41847
.9	.17592	.85997	.91885	2.1115	.46314
1.0	.21931	.95099	.90091	2.0631	.50413
1.1	.26820	1.04003	.87926	2.0108	.54070
1.2	.32285	1.12671	.85370	1.95514	.57213
1.3	.38355	1.21063	.82411	1.89660	.59771
1.4	.45062	1.29140	.79049	1.83582	.61681
1.5	.52435	1.36860	.75292	1.77347	.62893
1.6	.60506	1.44186	.71162	1.71028	.63369
1.7	.69306	1.51081	.66694	1.64699	.63091
1.8	.78861	1.57515	.61936	1.58435	.62064
1.9	.89197	1.63461	.56948	1.52310	.60312
2.0	1.00330	1.68899	.51803	1.46395	.57885
2.1	1.12273	1.73819	.46582	1.40753	.54858
2.2	1.25028	1.78216	.41373	1.35440	.51322
2.3	1.38585	1.82097	.36266	1.30502	.47386
2.4	1.52927	1.85476	.31349	1.25972	.43170
2.5	1.68020	1.88376	.26703	1.21873	.38797
2.6	1.83824	1.90827	.22397	1.18214	.34389
2.7	2.0029	1.92868	.18484	1.14993	.30059
2.8	2.1735	1.94539	.15002	1.12197	.25905
2.9	2.3494	1.95883	.11966	1.09803	.22011
3.0	2.5300	1.96947	.09376	1.07784	.18435
3.1	2.7147	1.97773	. 7213	1.06104	.15220
3.2	2.9026	1.98402	. 5447	1.04727	.12385
3.3	3.0934	1.98874	. 4036	1.03615	.09932
3.4	3.2863	1.99220	. 2934	1.02728	. 7850

TABLE B-2 (cont'd)

(i) Case A, $M_0 = 2.8$, Adiabatic (cont'd)

η	f_1	F_1	F_1'	θ_1	$-\theta_1'$
3.5	3.4811	1.99469	0.02092	1.02033	0.06115
3.6	3.6772	1.99645	. 1463	1.01495	. 4693
3.7	3.8744	1.99767	. 1003	1.01085	. 3550
3.8	4.0724	1.99850	. 0674	1.00777	. 2646
3.9	4.2709	1.99906	. 444	1.00549	. 1944
4.0	4.4700	1.99942	. 287	1.00383	. 1407
4.1	4.6693	1.99965	. 182	1.00263	. 1003
4.2	4.8688	1.99979	. 113	1.00179	. 0705
4.3	5.0685	1.99988	. 069	1.00120	. 488
4.4	5.2683	1.99993	. 41	1.00079	. 333
4.5	5.4682	1.99997	. 24	1.00052	. 224
4.6	5.6681	1.99998	. 14	1.00033	. 148
4.7	5.8680	1.99999	. 08	1.00021	. 097
4.8	6.0680	2.0000	. 4	1.00013	. 62
4.9	6.2680	2.0000	. 2	1.00008	. 39
5.0	6.4679	2.0000	. 1	1.00005	. 25
5.1	6.6679	2.0000	. 1	1.00003	. 15
5.2	6.8679	2.0000	. 0	1.00002	. 09
5.3	7.0679	2.0000	. 0	1.00001	. 5
5.4	7.2679	2.0000	. 0	1.00001	. 3
5.5	7.4679	2.0000	. 0	1.00001	. 2
5.6	7.6679	2.0000	. 0	1.00001	. 1
5.7	7.8679	2.0000	. 0	1.00000	. 1
5.8	8.0679	2.0000	. 0	1.00000	. 0
5.9	8.2679	2.0000	. 0	1.00000	. 0
6.0	8.4679	2.0000	. 0	1.00000	. 0

TABLE B-2 (cont'd)

(1) Case A, $M_0 = 2.8$, Adiabatic (cont'd)

η	f_2	F_2	F_2'	θ_2	θ_2'
0.0	0.0000	0.0000	7.806	0.8647	0.0000
.1	. 166	.8206	8.605	.8803	.2893
.2	. 686	1.7205	9.390	.9182	.4475
.3	.1597	2.698	10.148	.9655	.4776
.4	.2937	3.749	10.864	1.0096	.3853
.5	.4745	4.869	11.522	1.0387	.1795
.6	.7066	6.051	12.102	1.0421	- .1272
.7	.9946	7.286	12.589	1.0105	- .5179
.8	1.3440	8.565	12.962	0.9364	- .9720
.9	1.7603	9.874	13.205	.8148	-1.4647
1.0	2.250	11.201	13.301	.6431	-1.9676
1.1	2.820	12.529	13.234	.4220	-2.450
1.2	3.477	13.842	12.994	.1549	-2.879
1.3	4.229	15.122	12.574	- .1511	-3.224
1.4	5.084	16.350	11.971	- .4860	-3.453
1.5	6.050	17.510	11.191	- .8370	-3.543
1.6	7.134	18.583	10.244	-1.1892	-3.473
1.7	8.342	19.554	9.150	-1.5260	-3.234
1.8	9.680	20.41	7.937	-1.8304	-2.827
1.9	11.149	21.14	6.639	-2.086	-2.263
2.0	12.749	21.74	5.297	-2.278	-1.5641
2.1	14.475	22.20	3.956	-2.396	-0.7637
2.2	16.320	22.53	2.663	-2.429	.0976
2.3	18.269	22.73	1.4630	-2.376	.9742
2.4	20.31	22.83	0.3952	-2.235	1.8194
2.5	22.41	22.82	-0.5084	-2.014	2.590
2.6	24.55	22.73	-1.2271	-1.7212	3.247
2.7	26.71	22.58	-1.7525	-1.3693	3.765
2.8	28.87	22.39	-2.089	-0.9733	4.127
2.9	31.00	22.17	-2.252	- .5492	4.328
3.0	33.08	21.94	-2.267	- .1129	4.374
3.1	35.11	21.72	-2.165	.3209	4.280
3.2	37.06	21.51	-1.9788	.7393	4.071
3.3	38.95	21.32	-1.7405	1.1320	3.770
3.4	40.76	21.16	-1.4788	1.4914	3.408

TABLE B-2 (contd.)

(i) Case A, $M_0 = 2.8$, Adiabatic (contd.)

η	f_2	F_2	F_2'	θ_2	θ_2'
3.5	42.50	21.03	-1.2170	1.8125	3.010
3.6	44.19	20.92	-0.9718	2.093	2.601
3.7	45.82	20.83	- .7541	2.333	2.201
3.8	47.40	20.77	- .5694	2.534	1.8246
3.9	48.94	20.72	- .4186	2.699	1.4831
4.0	50.46	20.68	- .2999	2.832	1.1825
4.1	51.94	20.66	- .2095	2.937	0.9253
4.2	53.41	20.64	- .1428	3.019	.7108
4.3	54.86	20.63	- .0949	3.081	.5363
4.4	56.30	20.62	- . 616	3.127	.3974
4.5	57.73	20.61	- . 390	3.161	.2894
4.6	59.16	20.61	- . 241	3.186	.2071
4.7	60.58	20.61	- . 145	3.203	.1458
4.8	62.00	20.61	- . 085	3.215	.1009
4.9	63.42	20.61	- . 48	3.224	.0687
5.0	64.83	20.61	- . 26	3.230	. 460
5.1	66.25	20.61	- . 13	3.233	. 303
5.2	67.66	20.61	- . 06	3.236	. 197
5.3	69.07	20.61	- . 2	3.237	. 127
5.4	70.49	20.61	. 0	3.238	. 080
5.5	71.90	20.61	. 1	3.239	. 51
5.6	73.31	20.61	. 2	3.239	. 32
5.7	74.73	20.61	. 2	3.240	. 20
5.8	76.14	20.61	. 2	3.240	. 13
5.9	77.55	20.61	. 2	3.240	. 09
6.0	78.96	20.61	. 3	3.240	. 6

TABLE B-2 (cont'd)

(i) Case A, $M_0 = 2.8$, Adiabatic (cont'd)

η	f_3	F_3	F_3'	θ_3	θ_3'
0.0	0.000	0.000	-22.0	-0.918	0.00
.1	-. 39	-1.643	-10.86	- .784	2.68
.2	-.123	-2.16	0.506	- .388	5.20
.3	-.207	-1.534	12.10	.243	7.32
.4	-.243	0.263	23.9	1.055	8.82
.5	-.183	3.24	35.7	1.979	9.52
.6	.024	7.41	47.6	2.93	9.32
.7	.432	12.75	59.2	3.81	8.18
.8	1.099	19.25	70.6	4.54	6.16
.9	2.09	26.9	81.4	5.02	3.43
1.0	3.47	35.5	91.4	5.21	0.251
1.1	5.32	45.1	100.2	5.07	-3.02
1.2	7.72	55.5	107.6	4.61	-5.94
1.3	10.76	66.5	112.9	3.91	-8.06
1.4	14.51	78.0	115.7	3.05	-8.90
1.5	19.04	89.6	115.3	2.18	-8.10
1.6	24.4	100.9	111.1	1.490	-5.38
1.7	30.7	111.6	102.4	1.171	-0.675
1.8	37.8	121.2	88.7	1.416	5.87
1.9	45.8	129.2	69.5	2.39	13.85
2.0	54.4	135.0	45.2	4.21	22.7
2.1	63.5	138.1	16.53	6.93	31.5
2.2	72.7	138.2	-15.03	10.49	39.5
2.3	81.7	135.1	-47.2	14.77	45.7
2.4	90.1	128.8	-77.4	19.55	49.5
2.5	97.5	119.8	-102.6	24.6	50.3
2.6	103.4	108.5	-120.6	29.5	47.9
2.7	107.7	95.9	-129.9	34.0	42.4
2.8	110.2	82.9	-130.1	37.9	34.2
2.9	110.8	70.2	-122.1	40.8	24.0
3.0	109.7	58.6	-108.1	42.7	12.67
3.1	107.3	48.7	- 90.2	43.3	1.090
3.2	103.8	40.6	- 71.1	42.9	-9.92
3.3	99.7	34.4	- 52.8	41.4	-19.65
3.4	95.3	30.0	- 36.8	39.0	-27.6

TABLE B-2 (cont'd)

(i) Case A, $M_0 = 2.8$, Adiabatic (cont'd)

η	f_3	F_3	F_3'	θ_3	θ_3'
3.5	90.9	27.0	-23.8	36.0	-33.4
3.6	86.8	25.1	-14.03	32.4	-37.1
3.7	83.3	24.1	- 7.24	28.6	-38.6
3.8	80.5	23.6	- 2.90	24.7	-38.4
3.9	78.3	23.4	- 0.400	21.0	-36.6
4.0	76.8	23.5	.817	17.47	-33.7
4.1	76.0	23.6	1.232	14.27	-30.1
4.2	75.8	23.7	1.202	11.46	-26.1
4.3	76.1	23.8	0.971	9.06	-22.0
4.4	76.9	23.9	.687	7.05	-18.16
4.5	78.1	23.9	.430	5.42	-14.62
4.6	79.5	24.0	.231	4.11	-11.50
4.7	81.2	24.0	.095	3.10	- 8.85
4.8	83.1	24.0	. 11	2.33	- 6.67
4.9	85.1	24.0	- . 32	1.751	- 4.93
5.0	87.2	24.0	- . 49	1.329	- 3.56
5.1	89.3	24.0	- . 51	1.027	- 2.53
5.2	91.5	24.0	- . 45	0.814	- 1.762
5.3	93.8	24.0	- . 36	.668	- 1.204
5.4	96.1	24.0	- . 27	.568	- 0.807
5.5	98.4	24.0	- . 19	.502	- .532
5.6	100.7	24.0	- . 14	.459	- .344
5.7	103.0	24.0	- . 09	.431	- .219
5.8	105.3	24.0	- . 6	.414	- .137
5.9	107.6	24.0	- . 4	.403	- .085
6.0	109.9	24.0	- . 3	.396	- . 52

TABLE B-2 (cont'd)

(ii) Case B, $M_0 = 2.8$, Cooling

η	f_1	F_1	F_1'	θ_1	θ_1'
0.0	0.00000	0.00000	1.00882	2.0000	0.15119
.2	. 1002	.20173	1.00815	2.0184	.03311
.4	. 4000	.40299	1.00345	2.0133	-0.08425
.6	. 9030	.60258	0.99080	1.98493	- .19841
.8	.16182	.79853	.96649	1.93441	- .30506
1.0	.25597	.98818	.92731	1.86379	- .39848
1.2	.37458	1.16831	.87101	1.77633	- .47226
1.4	.51980	1.33539	.79686	1.67660	- .52037
1.6	.69391	1.48594	.70614	1.57018	- .53857
1.8	.89894	1.61697	.60248	1.46325	- .52570
2.0	1.13621	1.72645	.49181	1.36180	- .48452
2.2	1.40583	1.81372	.38163	1.27089	- .42162
2.4	1.70625	1.87965	.27971	1.19399	- .34620
2.6	2.0342	1.92658	.19253	1.13259	- .26810
2.8	2.385	1.95788	.12382	1.08636	- .19577
3.0	2.7533	1.97735	.07412	1.05352	- .13479
3.2	3.1342	1.98862	. 4119	1.03153	- .08750
3.4	3.5232	1.99467	. 2121	1.01764	- . 5357
3.6	3.9173	1.99767	. 1011	1.00937	- . 3093
3.8	4.3143	1.99905	. 0447	1.00472	- . 1684
4.0	4.7129	1.99964	. 183	1.00225	- . 0865
4.2	5.1122	1.99987	. 070	1.00102	- . 420
4.4	5.5119	1.99996	. 25	1.00044	- . 192
4.6	5.9118	1.99999	. 08	1.00018	- . 083
4.8	6.3118	2.0000	. 3	1.00007	- . 34
5.0	6.7117	2.0000	. 1	1.00002	- . 13
5.2	7.1117	2.0000	. 0	1.00001	- . 05
5.4	7.5117	2.0000	. 0	1.00000	- . 2
5.6	7.9117	2.0000	. 0	1.00000	- . 1
5.8	8.3117	2.0000	. 0	1.00000	- . 0
6.0	8.7117	2.0000	. 0	1.00000	- . 0

TABLE B-2 (cont'd)

(ii) Case B, $M_0 = 2.8$, Cooling (cont'd)

η	f_2	F_2	F_2'	θ_2	θ_2'
0.0	0.0000	0.0000	8.029	0.0000	0.7234
.2	.0843	1.7624	9.585	.1996	1.1809
.4	.3540	3.825	11.010	.4365	1.1028
.6	.8373	6.148	12.156	.6092	0.5556
.8	1.5683	8.659	12.866	.6361	- .3265
1.0	2.589	11.255	12.987	.4697	-1.3392
1.2	3.948	13.806	12.404	.1086	-2.228
1.4	5.700	16.166	11.073	- .3957	-2.732
1.6	7.895	18.189	9.057	- .9454	-2.659
1.8	10.568	19.756	6.549	-1.4163	-1.9476
2.0	13.714	20.80	3.862	-1.6893	-0.7133
2.2	17.275	21.31	1.3705	-1.6846	.7738
2.4	21.13	21.38	-0.5824	-1.3857	2.172
2.6	25.11	21.13	-1.7931	-0.8413	3.187
2.8	29.04	20.71	-2.255	- .1466	3.662
3.0	32.80	20.26	-2.138	.5888	3.607
3.2	36.31	19.874	-1.6984	1.2712	3.159
3.4	39.56	19.586	-1.1745	1.8402	2.506
3.6	42.61	19.398	-0.7204	2.273	1.8198
3.8	45.49	19.288	- .3962	2.575	1.2182
4.0	48.26	19.230	- .1969	2.770	0.7552
4.2	50.97	19.203	- .0890	2.887	.4350
4.4	53.64	19.191	- . 367	2.952	.2334
4.6	56.29	19.186	- . 139	2.986	.1170
4.8	58.92	19.184	- . 049	3.002	.0549
5.0	61.56	19.184	- . 16	3.010	. 241
5.2	64.19	19.184	- . 05	3.013	. 100
5.4	66.82	19.184	- . 1	3.014	. 039
5.6	69.45	19.184	- . 0	3.015	. 14
5.8	72.08	19.184	- . 0	3.015	. 05
6.0	74.71	19.184	- . 0	3.015	. 2

TABLE B-2 (cont'd)

(iii) Case C, $M_0 = 2.8$, Heating

η	f_1	F_1	F_1'	θ_1	$-\theta_1'$
0.0	0.00000	0.00000	0.89885	3.0000	0.27834
.1	. 151	. 8988	.89880	2.9698	.32520
.2	. 608	.17975	.89849	2.9350	.37197
.3	. 1379	.26956	.89762	2.8954	.41848
.4	. 2473	.35925	.89592	2.8513	.46452
.5	. 3903	.44871	.89309	2.8026	.50981
.6	. 5680	.53782	.88885	2.7494	.55399
.7	. 7820	.62642	.88290	2.6918	.59667
.8	.10340	.71433	.87494	2.6301	.63737
.9	.13259	.80133	.86471	2.5644	.67556
1.0	.16597	.88719	.85193	2.4951	.71068
1.1	.20378	.97163	.83635	2.4224	.74211
1.2	.24628	1.05436	.81778	2.3468	.76925
1.3	.29373	1.13508	.79602	2.2687	.79147
1.4	.34645	1.21345	.77098	2.1887	.80818
1.5	.40473	1.28916	.74260	2.1073	.81883
1.6	.46891	1.36186	.71089	2.0251	.82298
1.7	.53934	1.43123	.67598	1.94292	.82026
1.8	.61635	1.49696	.63806	1.86133	.81048
1.9	.70030	1.55875	.59745	1.78106	.79359
2.0	.79149	1.61637	.55454	1.70284	.76973
2.1	.89023	1.66960	.50985	1.62734	.73925
2.2	.99676	1.71830	.46398	1.55519	.70271
2.3	1.11126	1.76238	.41759	1.48697	.66085
2.4	1.23382	1.80183	.37141	1.42317	.61459
2.5	1.36445	1.83669	.32619	1.36417	.56500
2.6	1.50304	1.86712	.28264	1.31024	.51324
2.7	1.64936	1.89330	.24144	1.26155	.46050
2.8	1.80306	1.91550	.20317	1.21814	.40798
2.9	1.96369	1.93405	.16830	1.17991	.35677
3.0	2.1307	1.94929	.13715	1.14671	.30788
3.1	2.3035	1.96161	.10989	1.11823	.26211
3.2	2.4813	1.97139	.08651	1.09416	.22010
3.3	2.6636	1.97903	. 6689	1.07408	.18226
3.4	2.8496	1.98489	. 5078	1.05756	.14881

TABLE B-2 (cont'd)

(iii) Case C, $M_0 = 2.8$, Heating (cont'd)

η	f_1	F_1	F_1'	θ_1	$-\theta_1'$
3.5	3.0387	1.98929	0.03783	1.04417	0.11978
3.6	3.2304	1.99255	. 2765	1.03346	.09504
3.7	3.4242	1.99490	. 1983	1.02503	. 7433
3.8	3.6196	1.99657	. 1394	1.01847	. 5729
3.9	3.8162	1.99774	. 0961	1.01346	. 4352
4.0	4.0137	1.99854	. 650	1.00968	. 3258
4.1	4.2120	1.99907	. 431	1.00686	. 2403
4.2	4.4107	1.99942	. 280	1.00480	. 1747
4.3	4.6099	1.99965	. 178	1.00332	. 1251
4.4	4.8093	1.99979	. 111	1.00226	. 0883
4.5	5.0089	1.99988	. 068	1.00152	. 614
4.6	5.2087	1.99993	. 41	1.00101	. 421
4.7	5.4085	1.99996	. 24	1.00066	. 284
4.8	5.6084	1.99998	. 14	1.00042	. 189
4.9	5.8083	1.99999	. 08	1.00027	. 124
5.0	6.0083	2.0000	. 4	1.00017	. 080
5.1	6.2083	2.0000	. 2	1.00011	. 51
5.2	6.4082	2.0000	. 1	1.00006	. 32
5.3	6.6082	2.0000	. 1	1.00004	. 20
5.4	6.8082	2.0000	. 0	1.00002	. 12
5.5	7.0082	2.0000	. 0	1.00001	. 07
5.6	7.2082	2.0000	. 0	1.00001	. 4
5.7	7.4082	2.0000	. 0	1.00001	. 2
5.8	7.6082	2.0000	. 0	1.00000	. 1
5.9	7.8082	2.0000	. 0	1.00000	. 1
6.0	8.0082	2.0000	. 0	1.00000	. 0
6.1	8.2082	2.0000	. 0	1.00000	. 0
6.2	8.4082	2.0000	. 0	1.00000	. 0
6.3	8.6082	2.0000	. 0	1.00000	. 0
6.4	8.8082	2.0000	. 0	1.00000	. 0
6.5	9.0082	2.0000	. 0	1.00000	. 0

TABLE B-2 (cont'd)

(iii) Case C, $M_0 = 2.8$, Heating (cont'd)

η	f_2	F_2	F_2'	θ_2	θ_2'
0.0	0.0000	0.0000	7.550	0.0000	0.0679
.1	. 131	.7963	8.375	. 198	.3057
.2	. 547	1.6747	9.190	. 569	.4161
.3	.1282	2.634	9.984	. 988	.4025
.4	.2373	3.670	10.746	.1335	.2706
.5	.3858	4.781	11.462	.1493	.0285
.6	.5782	5.961	12.117	.1358	- .3128
.7	.8192	7.202	12.697	.0839	- .7387
.8	1.1137	8.497	13.187	- . 141	-1.2316
.9	1.4676	9.836	13.570	- .1639	-1.7699
1.0	1.8870	11.207	13.830	- .3688	-2.329
1.1	2.379	12.598	13.954	- .6295	-2.881
1.2	2.951	13.993	13.927	- .9439	-3.397
1.3	3.610	15.377	13.738	-1.3067	-3.845
1.4	4.367	16.735	13.379	-1.7096	-4.194
1.5	5.229	18.047	12.847	-2.141	-4.415
1.6	6.207	19.298	12.142	-2.587	-4.482
1.7	7.310	20.47	11.271	-3.032	-4.375
1.8	8.547	21.55	10.248	-3.456	-4.081
1.9	9.927	22.52	9.093	-3.842	-3.598
2.0	11.455	23.36	7.834	-4.170	-2.932
2.1	13.136	24.08	6.504	-4.423	-2.102
2.2	14.972	24.66	5.142	-4.586	-1.1383
2.3	16.958	25.11	3.791	-4.647	-0.0793
2.4	19.086	25.42	2.495	-4.600	1.0275
2.5	21.34	25.61	1.2970	-4.442	2.130
2.6	23.71	25.69	0.2333	-4.176	3.176
2.7	26.16	25.66	- .6654	-3.810	4.117
2.8	28.68	25.56	-1.3796	-3.357	4.910
2.9	31.22	25.39	-1.9011	-2.834	5.523
3.0	33.76	25.19	-2.234	-2.259	5.938
3.1	36.27	24.95	-2.393	-1.6531	6.149
3.2	38.73	24.71	-2.403	-1.0359	6.163
3.3	41.12	24.48	-2.294	-0.4263	6.001
3.4	43.43	24.26	-2.099	.1594	5.690

TABLE B-2 (cont'd)

(iii) Case C, $M_0 = 2.8$, Heating (cont'd)

η	f_2	F_2	F_2'	θ_2	$-\theta_2'$
3.5	45.65	24.06	-1.8495	0.7078	5.262
3.6	47.78	23.89	-1.5756	1.2092	4.754
3.7	49.83	23.74	-1.3005	1.6572	4.201
3.8	51.79	23.63	-1.0420	2.049	3.633
3.9	53.68	23.53	-0.8116	2.384	3.078
4.0	55.51	23.46	- .6151	2.666	2.556
4.1	57.29	23.41	- .4540	2.897	2.081
4.2	59.02	23.37	- .3266	3.084	1.6630
4.3	60.72	23.34	- .2291	3.232	1.3041
4.4	62.39	23.32	- .1568	3.347	1.0041
4.5	64.04	23.31	- .1047	3.434	0.7593
4.6	65.68	23.30	- .0683	3.500	.5641
4.7	67.30	23.30	- . 435	3.549	.4118
4.8	68.92	23.29	- . 270	3.584	.2955
4.9	70.53	23.29	- . 164	3.609	.2084
5.0	72.13	23.29	- . 097	3.626	.1446
5.1	73.74	23.29	- . 56	3.638	.0987
5.2	75.34	23.29	- . 31	3.646	. 662
5.3	76.93	23.29	- . 17	3.652	. 438
5.4	78.53	23.29	- . 08	3.655	. 285
5.5	80.13	23.29	- . 4	3.658	. 183
5.6	81.73	23.29	- . 1	3.659	. 116
5.7	83.32	23.29	. 0	3.660	. 073
5.8	84.92	23.29	. 1	3.661	. 45
5.9	86.52	23.29	. 1	3.661	. 28
6.0	88.11	23.29	. 1	3.661	. 18
6.1	89.71	23.29	. 1	3.661	. 11
6.2	91.31	23.29	. 2	3.661	. 08
6.3	92.90	23.29	. 2	3.661	. 6
6.4	94.50	23.29	. 2	3.662	. 4
6.5	96.10	23.29	. 2	3.662	. 4

TABLE B-2 (cont'd)

(iii) Case C, $M_0 = 2.8$, Heating (cont'd)

η	f_3	F_3	F_3'	θ_3	θ_3'
0.0	0.000	0.00	-35.8	0.000	4.54
.1	- .053	-2.90	-22.1	.618	7.80
.2	- .182	-4.41	- 8.10	1.553	10.83
.3	- .345	-4.51	6.30	2.77	13.39
.4	- .496	-3.14	21.0	4.21	15.23
.5	- .586	-0.293	36.0	5.79	16.18
.6	- .565	4.07	51.2	7.41	16.11
.7	- .377	9.95	66.4	8.97	14.94
.8	.040	17.35	81.6	10.36	12.69
.9	.752	26.3	96.6	11.48	9.50
1.0	1.835	36.6	111.1	12.24	5.56
1.1	3.37	48.5	125.0	12.58	1.218
1.2	5.45	61.6	137.8	12.48	-3.12
1.3	8.18	76.0	149.4	11.97	-6.95
1.4	11.66	91.4	159.0	11.12	-9.76
1.5	16.01	107.7	166.2	10.07	-11.02
1.6	21.3	124.6	170.2	8.99	-10.25
1.7	27.7	141.6	170.0	8.10	- 7.12
1.8	35.3	158.4	164.8	7.65	- 1.472
1.9	44.1	174.4	153.4	7.88	6.63
2.0	54.1	188.8	135.0	9.04	16.84
2.1	65.2	201	108.9	11.30	28.5
2.2	77.3	210	75.1	14.77	40.8
2.3	90.1	216	34.3	19.45	52.7
2.4	103.1	217	-11.47	25.3	62.9
2.5	115.9	214	- 59.4	32.0	70.5
2.6	127.9	205	-105.8	39.2	74.3
2.7	138.5	192.5	-146.3	46.7	73.9
2.8	147.2	176.3	-177.4	53.9	68.9
2.9	153.4	157.5	-196.2	60.3	59.6
3.0	157.0	137.5	- 20.2	65.7	46.6
3.1	157.9	117.6	-194.3	69.5	30.8
3.2	156.3	99.0	-176.7	71.8	13.38
3.3	152.6	82.5	-152.2	72.2	- 4.28
3.4	147.1	68.6	-124.4	70.9	-21.0

TABLE B-2 (cont'd)

(iii) Case C, $M_0 = 2.8$, Heating (cont'd)

η	f_3	F_3	F_3'	θ_3	θ_3'
3.5	140.5	57.6	-96.7	68.1	-35.6
3.6	133.5	49.2	-71.5	63.9	-47.5
3.7	126.4	43.1	-50.3	58.7	-56.2
3.8	119.7	39.0	-33.6	52.8	-61.4
3.9	113.8	36.3	-21.3	46.5	-63.5
4.0	108.8	34.6	-12.84	40.2	-62.6
4.1	104.9	33.6	- 7.31	34.1	-59.5
4.2	102.1	33.0	- 3.96	28.3	-54.6
4.3	100.2	32.8	- 2.06	23.2	-48.7
4.4	99.3	32.6	- 1.088	18.64	-42.2
4.5	99.3	32.5	- 0.636	14.75	-35.6
4.6	99.9	32.5	- .451	11.51	-29.3
4.7	101.1	32.4	- .384	8.87	-23.6
4.8	102.8	32.4	- .354	6.77	-18.56
4.9	104.9	32.4	- .326	5.13	-14.29
5.0	107.2	32.3	- .290	3.89	-10.78
5.1	109.8	32.3	- .246	2.95	- 7.96
5.2	112.5	32.3	- .199	2.27	- 5.77
5.3	115.3	32.3	- .154	1.783	- 4.10
5.4	118.2	32.2	- .115	1.439	- 2.86
5.5	121.2	32.2	- .083	1.201	- 1.954
5.6	124.2	32.2	- . 57	1.040	- 1.312
5.7	127.2	32.2	- . 39	0.932	- 0.866
5.8	130.2	32.2	- . 26	.862	- .561
5.9	133.3	32.2	- . 16	.816	- .358
6.0	136.3	32.2	- . 10	.788	- .224
6.1	139.4	32.2	- . 06	.770	- .138
6.2	142.5	32.2	- . 4	.759	- .084
6.3	145.6	32.2	- . 2	.753	- . 51
6.4	148.6	32.2	- . 2	.749	- . 31
6.5	151.7	32.2	- . 1	.746	- . 19

TABLE B-2 (cont'd)

(iv) Case D, $M_0 = 2$, Adiabatic

η	f_1	F_1	F_1'	θ_1	$-\theta_1'$
0.0	0.00000	0.00000	1.09758	1.68094	0.00000
.2	. 1309	.21947	1.09663	1.67381	.07125
.4	. 5265	.43827	1.08993	1.65250	.14159
.6	.11954	.65468	1.07182	1.61739	.20867
.8	.21505	.86588	1.03706	1.56949	.26878
1.0	.34079	1.06811	0.98150	1.51065	.31726
1.2	.49843	1.25694	.90299	1.44368	.34934
1.4	.68947	1.42781	.80230	1.37226	.36133
1.6	.91476	1.57666	.68378	1.30059	.35183
1.8	1.17407	1.70064	.55520	1.23285	.32255
2.0	1.46570	1.79873	.42661	1.17257	.27827
2.2	1.78632	1.87196	.30833	1.12209	.22583
2.4	2.1313	1.92328	.20849	1.08231	.17239
2.6	2.4953	1.95686	.13134	1.05280	.12384
2.8	2.8732	1.97730	.07684	1.03221	.08376
3.0	3.2603	1.98885	. 4166	1.01886	. 5337
3.2	3.6532	1.99489	. 2092	1.01026	. 3206
3.4	4.0495	1.99782	. 0972	1.00535	. 1815
3.6	4.4477	1.99913	. 419	1.00265	. 0969
3.8	4.8468	1.99968	. 167	1.00124	. 488
4.0	5.2464	1.99989	. 062	1.00055	. 232
4.2	5.6463	1.99996	. 22	1.00023	. 104
4.4	6.0462	1.99999	. 07	1.00009	. 044
4.6	6.4462	2.0000	. 2	1.00004	. 18
4.8	6.8462	2.0000	. 1	1.00001	. 07
5.0	7.2462	2.0000	. 0	1.00000	. 2
5.2	7.6462	2.0000	. 0	1.00000	. 1
5.4	8.0462	2.0000	. 0	1.00000	. 0
5.6	8.4462	2.0000	. 0	1.00000	. 0
5.8	8.8462	2.0000	. 0	1.00000	. 0
6.0	9.2462	2.0000	. 0	1.00000	. 0

TABLE B-2 (cont'd)

(iv) Case D, $M_0 = 2$, Adiabatic (cont'd)

η	f_2	F_2	F_2'	θ_2	θ_2'
0.0	0.0000	0.0000	4.034	0.6071	0.0000
.2	. 524	1.0403	6.356	.6865	.7200
.4	.2443	2.532	8.519	.8661	1.0065
.6	.6264	4.425	10.330	1.0630	0.9046
.8	1.2490	6.626	11.572	1.2083	.5104
1.0	2.162	9.002	12.047	1.2576	- .0253
1.2	3.412	11.384	11.617	1.2011	- .5165
1.4	5.036	13.587	10.270	1.0660	- .7854
1.6	7.049	15.441	8.162	0.9095	- .7210
1.8	9.432	16.823	5.618	.8001	- .3241
2.0	12.121	17.687	3.070	.7944	.2886
2.2	15.011	18.077	0.9319	.9176	.9323
2.4	17.978	18.104	- .5257	1.1576	1.4309
2.6	20.91	17.914	-1.2514	1.4735	1.6815
2.8	23.73	17.641	-1.3879	1.8134	1.6745
3.0	26.40	17.379	-1.1732	2.131	1.4722
3.2	28.92	17.177	-0.8329	2.396	1.1669
3.4	31.34	17.043	- .5154	2.597	0.8441
3.6	33.66	16.964	- .2832	2.737	.5616
3.8	35.93	16.923	- .1398	2.827	.3454
4.0	38.17	16.903	- .0625	2.880	.1971
4.2	40.39	16.895	- . 255	2.910	.1047
4.4	42.61	16.892	- . 095	2.925	.0519
4.6	44.81	16.891	- . 33	2.932	. 241
4.8	47.02	16.890	- . 11	2.935	. 104
5.0	49.22	16.890	- . 03	2.937	. 043
5.2	51.42	16.890	- . 1	2.937	. 16
5.4	53.63	16.890	- . 0	2.937	. 06
5.6	55.83	16.890	- . 0	2.937	. 2
5.8	58.03	16.890	- . 0	2.937	. 1
6.0	60.24	16.890	. 0	2.937	. 0

TABLE B-2 (cont'd)

(v) Case E, $M_0 = 2$, Cooling

η	f_1	F_1	F_1'	θ_1	θ_1'
0.0	0.00000	0.00000	1.14297	1.44500	0.12277
.2	. 1568	.22853	1.14177	1.46182	. 4542
.4	. 6248	.45624	1.13347	1.46320	- . 3131
.6	.14058	.68099	1.11127	1.44949	- .10500
.8	.25074	.89941	1.06921	1.42167	- .17156
1.0	.39403	1.10706	1.00301	1.38168	- .22584
1.2	.57157	1.29890	0.91122	1.33249	- .26280
1.4	.78408	1.46998	.79616	1.27796	- .27890
1.6	1.03144	1.61623	.66437	1.22238	- .27345
1.8	1.31224	1.73526	.52585	1.16984	- .24915
2.0	1.62354	1.82689	.39227	1.12360	- .21166
2.2	1.96097	1.89320	.27423	1.08559	- .16805
2.4	2.3193	1.93808	.17882	1.05634	- .12499
2.6	2.6931	1.96637	.10839	1.03523	- .08725
2.8	3.0776	1.98294	.06093	1.02092	- . 5726
3.0	3.4690	1.99192	. 3172	1.01179	- . 3538
3.2	3.8644	1.99644	. 1528	1.00631	- . 2060
3.4	4.2621	1.99854	. 0682	1.00320	- . 1131
3.6	4.6610	1.99944	. 282	1.00154	- . 0585
3.8	5.0605	1.99980	. 108	1.00070	- . 286
4.0	5.4603	1.99993	. 039	1.00030	- . 132
4.2	5.8602	1.99998	. 13	1.00012	- . 057
4.4	6.2602	1.99999	. 04	1.00005	- . 24
4.6	6.6602	2.0000	. 1	1.00002	- . 09
4.8	7.0602	2.0000	. 0	1.00001	- . 3
5.0	7.4602	2.0000	. 0	1.00000	- . 1
5.2	7.8602	2.0000	. 0	1.00000	- . 0
5.4	8.2602	2.0000	. 0	1.00000	- . 0
5.6	8.6602	2.0000	. 0	1.00000	- . 0
5.8	9.0602	2.0000	. 0	1.00000	- . 0
6.0	9.4602	2.0000	. 0	1.00000	- . 0

TABLE B-2 (cont'd)

(v) Case E, $M_0 = 2$, Cooling (cont'd)

η	f_2	F_2	F_2'	θ_2	θ_2'
0.0	0.0000	0.0000	4.514	0.0000	0.4449
.2	.0712	1.1284	6.754	.1690	1.1688
.4	.3173	2.689	8.803	.4375	1.4441
.6	.7838	4.622	10.442	.7199	1.3206
.8	1.5138	6.823	11.437	.9462	0.9055
1.0	2.548	9.141	11.590	1.0735	.3628
1.2	3.921	11.396	10.800	1.0956	- .1152
1.4	5.651	13.401	9.120	1.0431	- .3613
1.6	7.732	15.000	6.789	0.9723	- .2934
1.8	10.117	16.099	4.207	.9445	.0525
2.0	12.724	16.695	1.8269	1.0029	.5407
2.2	15.442	16.867	0.0176	1.1588	1.0009
2.4	18.161	16.750	-1.0519	1.3925	1.3011
2.6	20.80	16.490	-1.4370	1.6653	1.3895
2.8	23.30	16.204	-1.3473	1.9362	1.2902
3.0	25.68	15.964	-1.0299	2.174	1.0711
3.2	27.93	15.792	-0.6775	2.362	0.8076
3.4	30.10	15.686	- .3930	2.499	.5585
3.6	32.21	15.628	- .2039	2.589	.3563
3.8	34.28	15.599	- .0955	2.645	.2106
4.0	36.34	15.586	- . 407	2.677	.1158
4.2	38.38	15.581	- . 158	2.694	.0593
4.4	30.42	15.579	- . 057	2.702	. 284
4.6	42.45	15.578	- . 19	2.706	. 127
4.8	44.48	15.578	- . 06	2.708	. 053
5.0	46.51	15.578	- . 2	2.709	. 21
5.2	48.55	15.578	- . 0	2.709	. 08
5.4	50.58	15.578	- . 0	2.709	. 3
5.6	52.61	15.578	- . 0	2.709	. 1
5.8	54.64	15.578	- . 0	2.709	. 0
6.0	56.67	15.578	- . 0	2.709	. 0

TABLE B-2 (cont'd)

(vi) Case F, $M_0 = 2$, Heating

η	f_1	F_1	F_1'	θ_1	$-\theta_1'$
0.0	0.00000	0.00000	1.02125	2.1670	0.22692
.2	. 957	.20422	1.02060	2.1154	.28852
.4	. 3900	.40798	1.01600	2.0517	.34888
.6	. 8962	.61008	1.00338	1.97612	.40580
.8	.16311	.80853	0.97873	1.88979	.45610
1.0	.26147	1.00053	.93845	1.79439	.49578
1.2	.38691	1.18269	.87995	1.69247	.52060
1.4	.54174	1.35123	.80232	1.58740	.52675
1.6	.72803	1.50243	.70703	1.48317	.51191
1.8	.94729	1.63313	.59831	1.38404	.47610
2.0	1.19994	1.74130	.48297	1.29394	.42226
2.2	1.48495	1.82643	.36945	1.21597	.35598
2.4	1.79957	1.88975	.26615	1.15190	.28449
2.6	2.1396	1.93400	.17958	1.10203	.21511
2.8	2.4998	1.96290	.11299	1.06533	.15367
3.0	2.8749	1.98048	.06606	1.03980	.10364
4.2	3.2602	1.99041	. 3582	1.02305	.06596
3.4	3.6521	1.99561	. 1799	1.01268	. 3960
3.6	4.0478	1.99813	. 0836	1.00661	. 2242
3.8	4.4457	1.99925	. 360	1.00327	. 1198
4.0	4.8447	1.99972	. 144	1.00153	. 0604
4.2	5.2442	1.99990	. 054	1.00068	. 287
4.4	5.6440	1.99997	. 19	1.00029	. 129
4.6	6.0439	1.99999	. 06	1.00011	. 055
4.8	6.4439	2.0000	. 2	1.00004	. 22
5.0	6.8439	2.0000	. 1	1.00002	. 08
5.2	7.2439	2.0000	. 0	1.00001	. 3
5.4	7.6439	2.0000	. 0	1.00000	. 1
5.6	8.0439	2.0000	. 0	1.00000	. 0
5.8	8.4439	2.0000	. 0	1.00000	. 0
6.0	8.8439	2.0000	. 0	1.00000	. 0

TABLE B-2 (cont'd)

(vi) Case F, $M_0 = 2$, Heating (cont'd)

η	f_2	F_2	F_2'	θ_2	θ_2'
0.0	0.0000	0.0000	3.239	0.0000	0.3602
.2	. 378	.8969	5.721	.1452	1.0168
.4	.1830	2.280	8.078	.3775	1.2360
.6	.4845	4.109	10.154	.6133	1.0621
.8	.9945	6.311	11.766	.7818	0.5829
1.0	1.7687	8.772	12.724	.8351	- .0636
1.2	2.866	11.346	12.865	.7571	- .6978
1.4	4.343	13.857	12.100	.5697	-1.1278
1.6	6.249	16.127	10.465	.3303	-1.1979
1.8	8.605	17.998	8.151	.1193	-0.8423
2.0	11.396	19.364	5.496	.0182	- .1200
2.2	14.551	20.20	2.919	. 843	.7943
2.4	17.953	20.56	0.8084	.3333	1.6686
2.6	21.45	20.57	- .5999	.7355	2.297
2.8	24.91	20.37	-1.2796	1.2288	2.570
3.0	28.23	20.09	-1.3855	1.7405	2.490
3.2	31.36	19.835	-1.1570	2.209	2.153
3.4	34.31	19.636	-0.8147	2.594	1.6873
3.6	37.11	19.505	- .5010	2.884	1.2102
3.8	39.80	19.429	- .2738	3.084	0.7996
4.0	42.42	19.389	- .1345	3.211	.4889
4.2	44.99	19.370	- .0598	3.286	.2776
4.4	47.54	19.362	- . 243	3.328	.1468
4.6	50.08	19.359	- . 090	3.349	.0725
4.8	52.61	19.358	- . 31	3.359	. 335
5.0	55.13	19.358	- . 10	3.363	. 145
5.2	57.66	19.358	- . 03	3.365	. 059
5.4	60.18	19.358	- . 1	3.366	. 23
5.6	62.71	19.358	- . 0	3.366	. 08
5.8	65.23	19.358	- . 0	3.366	. 3
6.0	67.76	19.358	- . 0	3.367	. 1

TABLE B-3

Values of β_n, t_n, α_n, and k_n

Case	β_1	β_2	β_3	t_1	t_2	t_3
A	11.3357	47.20	198.4	3.2388	0.373	32.4
B	10.5533	38.29		3.0152	-0.424	
C	12.8111	61.16	310	3.6603	0.727	51.2
D	10.2810	57.72		2.9374	5.71	
E	9.4821	46.50		2.7092	4.11	
F	11.7828	76.93		3.3665	7.81	

Case	α_1	α_2	α_3	k_1	k_2	k_3
A	8.0969	20.60	96.2	-1.03277	-0.652	-11.02
B	7.5380	15.986		-0.96149	-0.327	
C	9.1508	26.94	153.4	-1.16719	-0.913	-17.41
D	7.3435	30.45		-1.83589	-5.25	
E	6.7729	24.04		-1.69324	-4.00	
F	8.4163	40.78		-2.1041	-7.10	

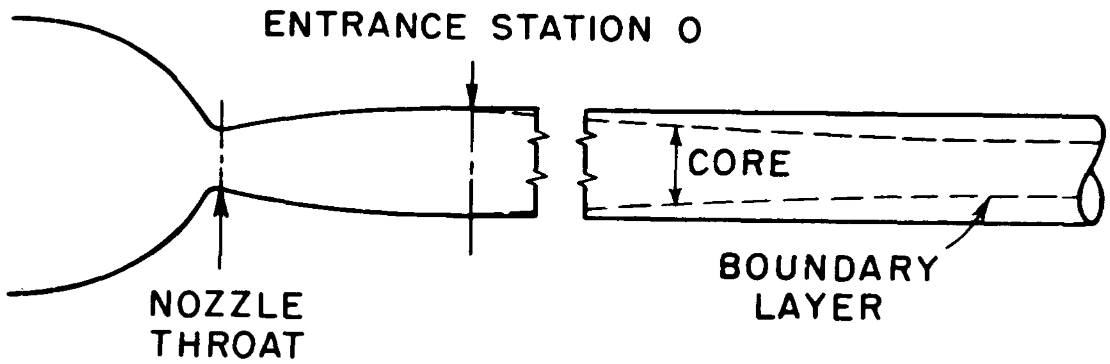


FIG. 1-MODEL FOR TUBE FLOW

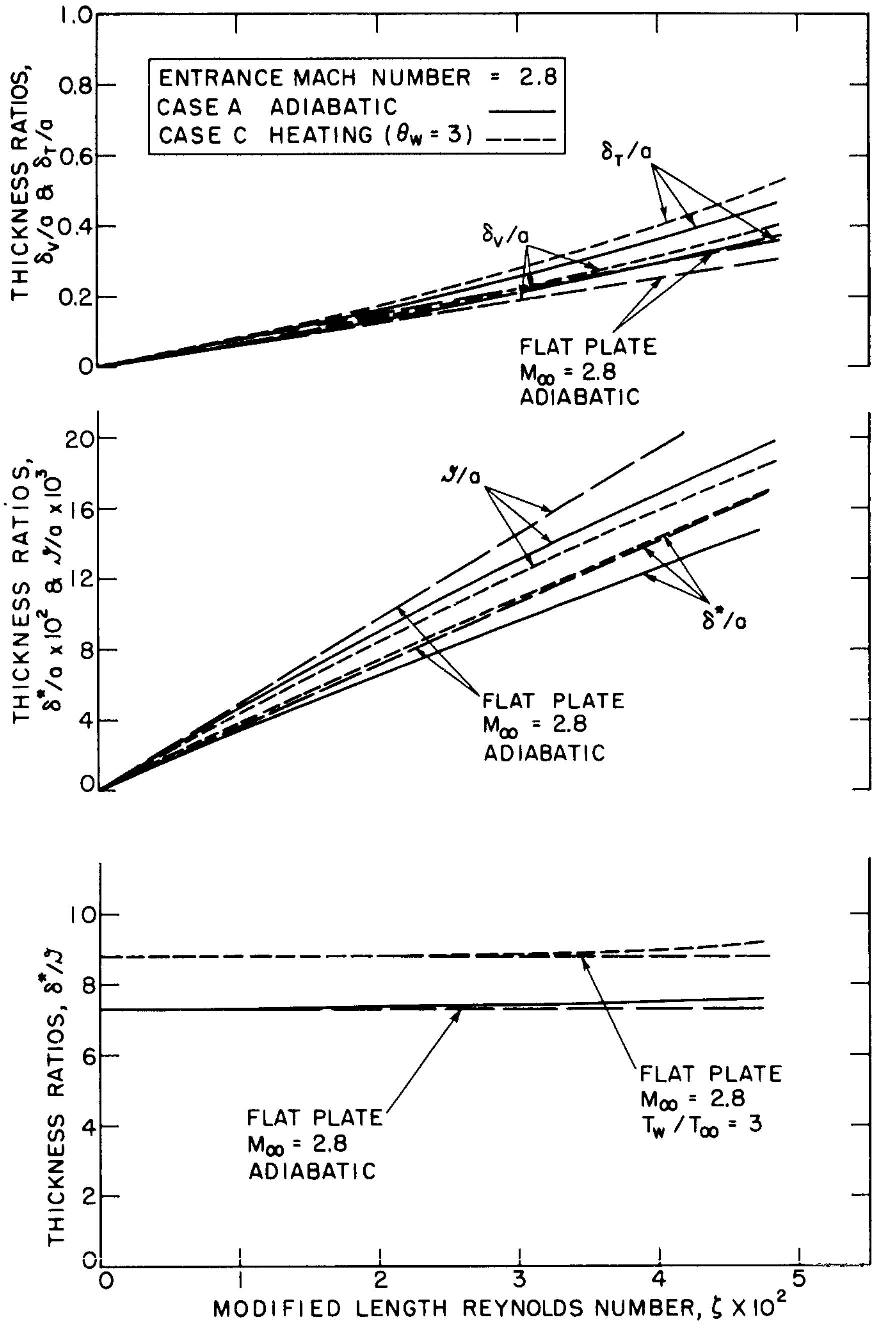


FIG. 2 - VARIATION OF BOUNDARY-LAYER THICKNESS, DISPLACEMENT THICKNESS, AND MOMENTUM THICKNESS ALONG TUBE LENGTH

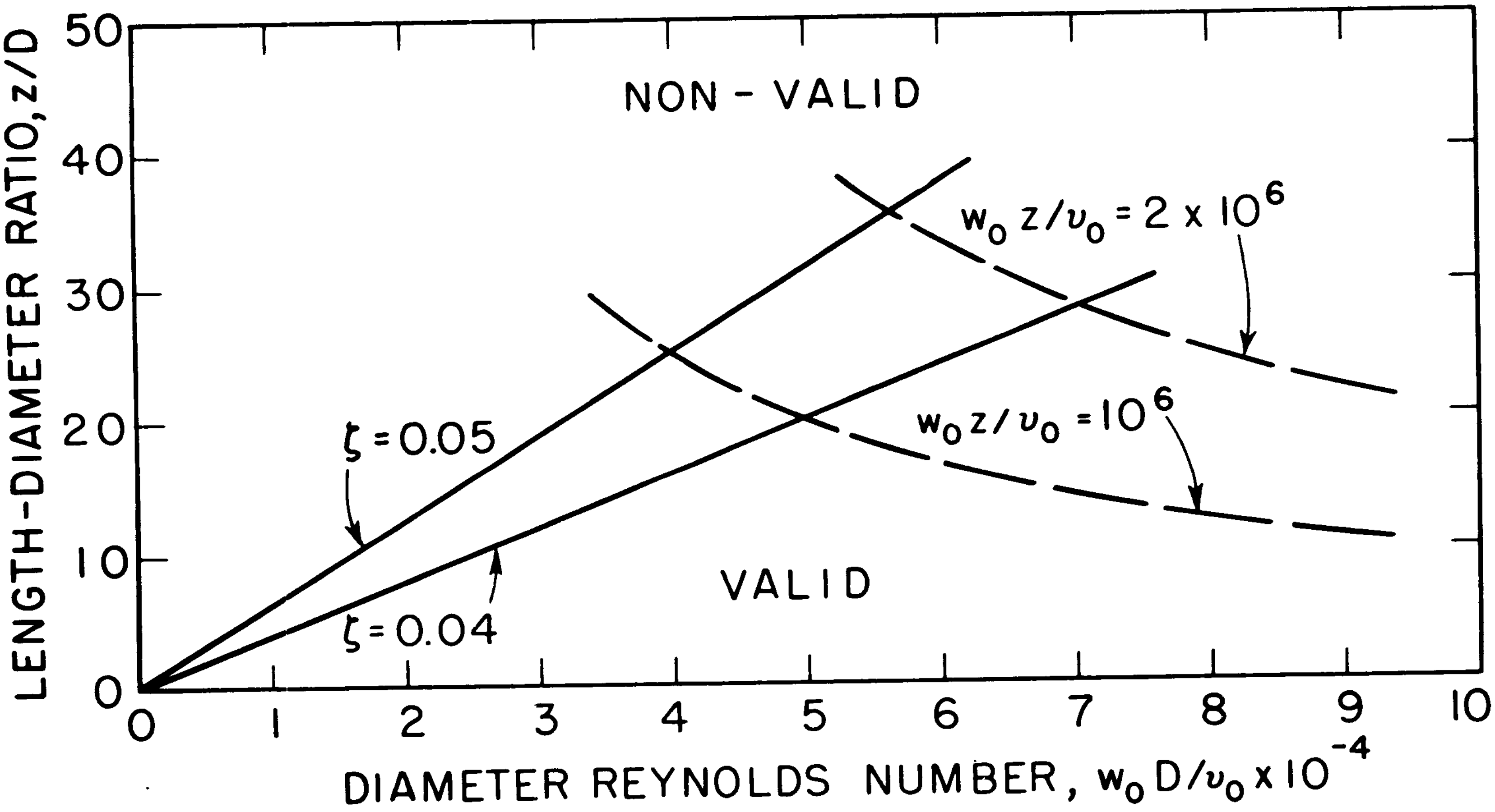


FIG. 3 - REGION OF VALIDITY OF PRESENT ANALYSIS

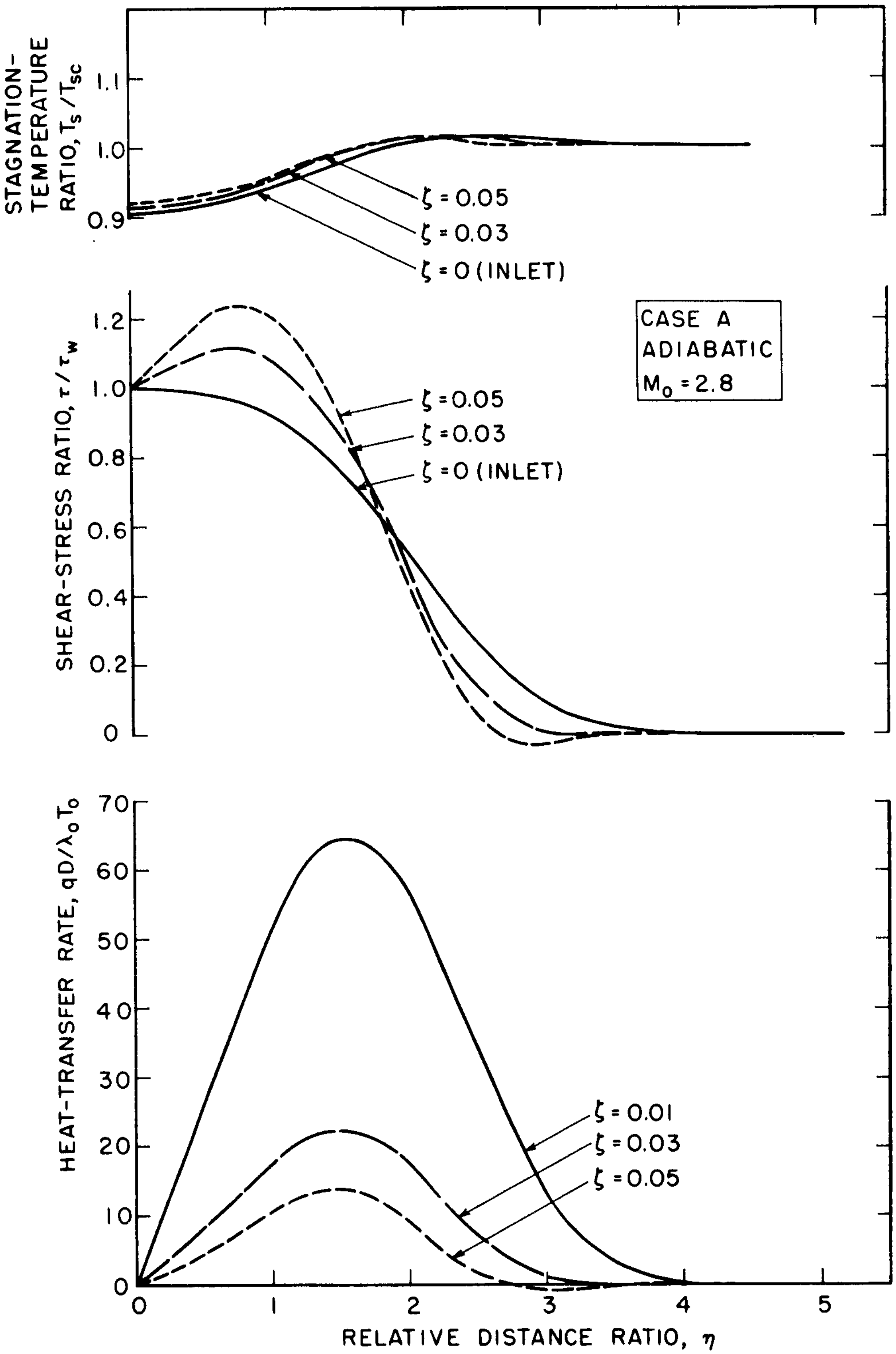


FIG. 4 - PROFILES OF STAGNATION TEMPERATURE, SHEAR STRESS, AND HEAT-TRANSFER RATE

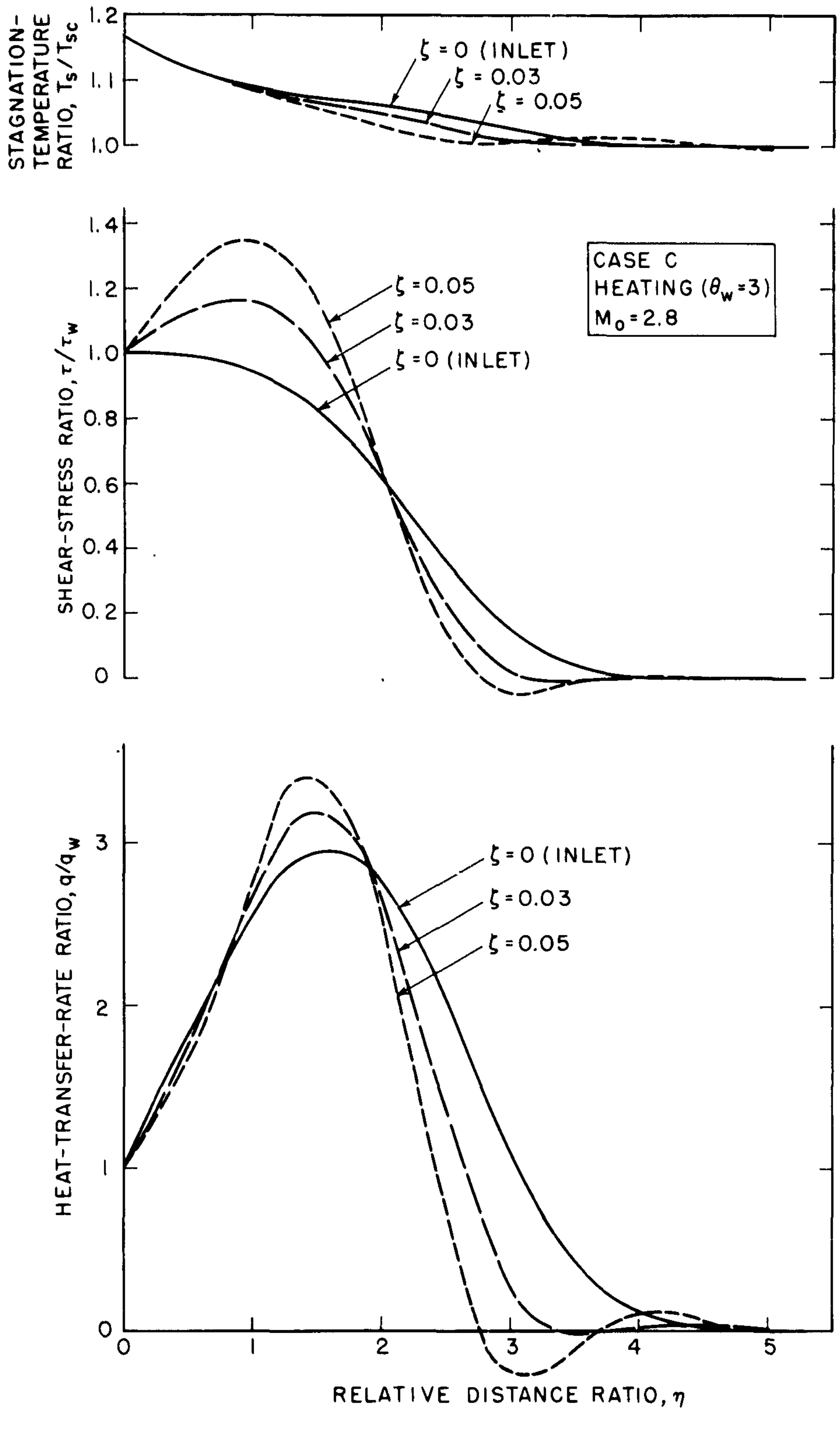


FIG. 5-PROFILES OF STAGNATION TEMPERATURE, SHEAR STRESS, AND HEAT-TRANSFER RATE

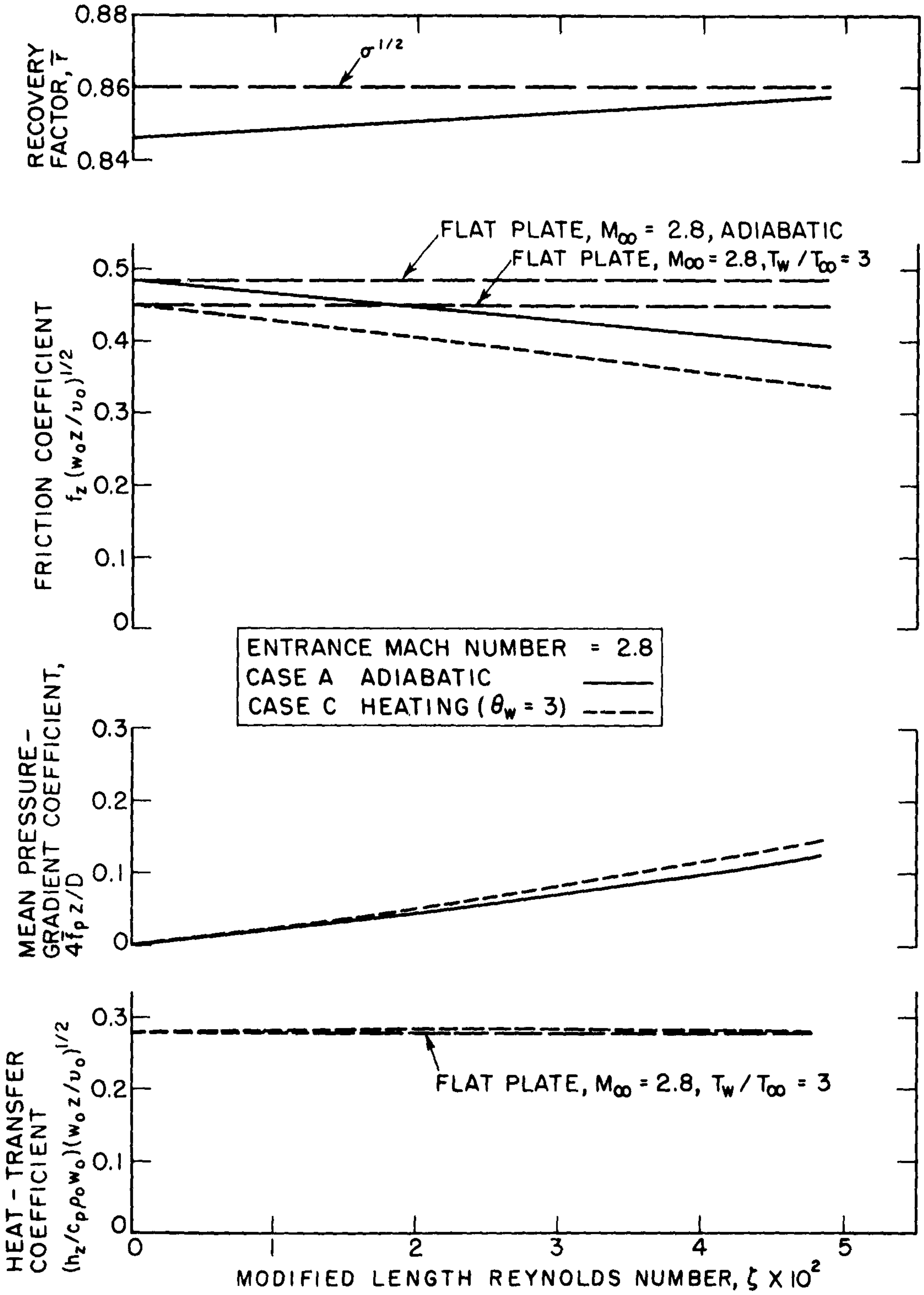


FIG. 6 - VARIATION OF RECOVERY FACTOR, FRICTION COEFFICIENT, MEAN PRESSURE - GRADIENT COEFFICIENT, AND HEAT-TRANSFER COEFFICIENT ALONG TUBE LENGTH

REVISED DISTRIBUTION LIST FOR UNCLASSIFIED
TECHNICAL REPORTS ISSUED UNDER
CONTRACT N5ori-07897

Chief of Naval Research
Department of the Navy
Washington 25, D. C.
Attn: Code 438 (2)

Commanding Officer
Office of Naval Research
Branch Office
150 Causeway Street
Boston 10, Massachusetts (2)

Commanding Officer
Office of Naval Research
Branch Office
346 Broadway
New York 13, New York (1)

Commanding Officer
Office of Naval Research
Branch Office
Tenth Floor
The John Crerar Library Bldg.
86 East Randolph Street
Chicago 1, Illinois (1)

Commanding Officer
Office of Naval Research
Branch Office
1000 Geary Street
San Francisco 1, California (1)

Commanding Officer
Office of Naval Research
Branch Office
1030 E. Green Street
Pasadena 1, California (1)

Commanding Officer
Office of Naval Research
Navy #100, FPO
New York, New York (2)

Chief, Bureau of Aeronautics
Department of the Navy
Washington 25, D. C.
Attn: Research Division (1)

Chief, Bureau of Ordnance
Department of the Navy
Washington 25, D. C.
Attn: Code Re9a (1)

Commanding Officer
Naval Ordnance Laboratory
White Oak, Silver Spring, Md.
Attn: Dr. R. K. Lobb (1)

Chief, Bureau of Ships
Department of the Navy
Washington 25, D. C.
Attn: Research Division (1)

Commanding Officer and Director
David Taylor Model Basin
Washington 7, D. C.
Attn: Aeromechanics Division (1)
Hydrodynamics Division (1)

Directorate of Intelligence
Headquarters, U.S. Air Force
National Defense Building
Washington 25, D. C.
Attn: Doc. and Dissem. Br.(AFOIR-DD) (1)

Armed Services Technical
Information Agency
Document Service Center
Knott Building
Dayton 2, Ohio (5)

Commanding General, Hdqs.
Air Research and Development Command
Office of Scientific Research
P. O. Box 1395
Baltimore, 18, Maryland
Attn: Fluid Mechanics Div. (1)

Director, Research and Development
Department of the Army
National Defense Building
Washington 25, D. C. (1)

Commanding General
Office of Ordnance Research
Department of the Army
Washington 25, D. C. (1)

Ballistic Research Laboratories
Department of the Army
Aberdeen Proving Ground, Maryland
Attn: Mr. J. Sternberg (1)

Director of Research
National Advisory Committee
for Aeronautics
1724 F Street, Northwest
Washington 25, D. C. (1)

Director
National Advisory Committee
for Aeronautics
Langley Aeronautical Laboratory
Langley Field, Virginia (1)

Director
National Advisory Committee
for Aeronautics
Langley Aeronautical Laboratory
Langley Field, Virginia
Attn: Dr. A. Busemann (1)

Director
National Advisory Committee
for Aeronautics
Ames Aeronautical Laboratory
Moffett Field, California (1)

Director
National Advisory Committee
for Aeronautics
Ames Aeronautical Laboratory
Moffett Field, California
Attn: Dr. D. R. Chapman (1)

Director
National Advisory Committee
for Aeronautics
Lewis Flight Propulsion Laboratory
21000 Brookpark Road
Cleveland 11, Ohio (1)

Director
National Advisory Committee
for Aeronautics
Lewis Flight Propulsion Laboratory
21000 Brookpark Road
Cleveland 11, Ohio
Attn: Dr. J. C. Evvard (1)

The Johns Hopkins University
Department of Aeronautical Engineering
Baltimore 18, Maryland
Attn: Dr. F. H. Clauser (1)

The Johns Hopkins University
Applied Physics Laboratory
8621 Georgia Avenue
Silver Spring, Maryland
Attn: Dr. F. N. Frenkiel (1)

University of Michigan
Department of Aeronautical Engineering
East Engineering Building
Ann Arbor, Michigan
Attn: Dr. Arnold Kuethe (1)
Prof. E. W. Conlon (1)

California Institute of Technology
Guggenheim Aeronautical Laboratory
Pasadena 4, California
Attn: Dr. Hans W. Liepmann (1)
Prof. Lester Lees (1)
Dr. P. A. Lagerstrom (1)

Director
National Bureau of Standards
Department of Commerce
Washington 25, D. C.
Attn: Dr. G. B. Schubauer (1)

Polytechnic Institute of Brooklyn
Department of Aeronautical Engineering
and Applied Mechanics
99 Livingston Street
Brooklyn 2, New York
Attn: Dr. A. Ferri (1)

Brown University
Division of Engineering
Providence 12, Rhode Island
Attn: Dr. P. F. Maeder (1)

California Institute of Technology
Jet Propulsion Laboratory
Pasadena 4, California
Attn: Mr. F. E. Goddard (1)

University of California
Engineering Research Projects
Berkeley 4, California
Attn: Dr. S. A. Schaaf (1)

University of California
Department of Engineering
Los Angeles 24, California
Attn: Dean L.M.K. Boelter (1)

Cornell Aeronautical Laboratory
4455 Genessee Street
Buffalo 21, New York
Attn: Mr. A. H. Flax (1)

Cornell University
Graduate School of Aeronautical
Engineering
Ithaca, New York
Attn: Dr. W. R. Sears (1)

University of Delaware
Department of Chemical Engineering
Newark, Delaware
Attn: Dr. Kurt Wohl (1)

Harvard University
Department of Applied Physics
and Engineering Science
Cambridge 38, Massachusetts
Attn: Prof. H. W. Emmons (1)

Massachusetts Institute of Technology
Guided Missiles Committee
Cambridge 39, Massachusetts
Attn: Supervisor, Tech. Reports (1)

Massachusetts Institute of Technology
Department of Aeronautical Engineering
Cambridge 39, Massachusetts
Attn: Dr. M. Finston (1)

Massachusetts Institute of Technology
Department of Mechanical Engineering
Cambridge 39, Massachusetts
Attn: Prof. J. H. Keenan (1)
Prof. A. H. Shapiro (1)

Princeton University
Department of Aeronautical Engineering
Princeton, New Jersey
Attn: Prof. S. M. Bogdonoff (1)

Office of Naval Research (15)
Branch Office
Navy Number 100
Fleet Post Office
New York, New York

Purdue University
School of Mechanical Engineering
Lafayette, Indiana
Attn: Dr. N. J. Zucrow (1)

University of Texas
Defense Research Laboratory
500 East 24th Street
Austin, Texas
Attn: Dr. M. J. Thompson (1)

Rensselaer Polytechnic Institute
Department of Aeronautical Engineering
Troy, New York
Attn: Dr. R. P. Harrington (1)

University of Maryland
Institute for Fluid Dynamics
and Applied Mathematics
College Park, Maryland
Attn: Dr. S. I. Pai (1)

Case Institute of Technology
Department of Mechanical Engineering
Cleveland, Ohio
Attn: Prof. G. Kuerti (1)

Stanford University
Department of Mechanical Engineering
Stanford, California
Attn: Prof. A. L. London (1)